

Computer Adoption and the Changing Labor Market*

Miguel Morin[†] Filip Rozsygal[‡]

February 15, 2018

Abstract

This paper examines computer adoption as a theoretical explanation for changes in the US labor market in recent decades. When computers become cheap and competitive compared to labor services, they diffuse more rapidly in the conventional mechanism of capital-labor substitution. We build a quantitative model which accounts for recent structural changes with this trend of automation: employment shifts away from routine occupations and the labor share of income declines. With hiring costs, firms entering a recession “front-load” the destruction of routine jobs, which accounts for recent cyclical changes of the labor market: routine job losses are concentrated in recessions and the ensuing recoveries are jobless. This paper also tests this labor demand mechanism against the labor supply mechanism of Jaimovich and Siu (2012b): computer adoption predicts job layoffs but not job quits among the unemployed.

Keywords: computers; routinization; jobless recoveries; business cycles; labor market.

JEL codes: E22, E24, E32, J3.

*We thank Suresh Naidu, Ricardo Reis, Bernard Salanié, and Jón Steinsson, Martín Uribe, Charles Calomiris, Christopher Conlon, Michael Johannes, Stephanie Schmitt-Grohé, and three anonymous referees for their useful comments. Morin gratefully acknowledges *Fundação para a Ciência e Tecnologia* for the financial support of his doctoral dissertation with grant SFRH / BD / 64060 / 2009.

[†]Alan Turing Institute, mmorin@turing.ac.uk, 96 Euston Road, London NW1 2DB, UK.

[‡]Danmarks Nationalbank, firo@nationalbanken.dk, Havnegade 5, 1093 Kobenhavn K, Denmark

1 Introduction

Over several decades, the US labor market has changed both in terms of short run cyclical dynamics as well as composition over longer horizon. Employment has shifted away from routine occupations since 1990. This shift away has been concentrated in recessions and the ensuing recoveries have been jobless, i.e. employment recovers much slower than output. At the same time, the cost of technologies which are designed to perform repetitive tasks which can be automated, has been falling exponentially. In this paper, we use a quantitative model to shed light on the link between the two trends.

We argue that the long term changes in the supply side of the economy can have exactly the implications for the business cycle fluctuations which we see from the US data. To show this, we build a quantitative model with the two distinct features. First, the model is non-stationary: the cost of producing particular type of capital is (stochastically) falling over time. Agents are aware of this trend and take it to the account when making choices. Second, we assume that the changes in employment are subject to recruiting costs. The interaction of the two components generates the results. Early, when the costs of the technology which replaces the routine workers is high, these workers are fired in the recessions but re-hired when the productivity bounces back. However, at some point, the cost of computers becomes low enough that firms start using them instead of routine workers, who get fired in recessions and never hired back.

We solve for the full transitional dynamics in a fully rational model. Maliar et al. (2015) provide a general method how to solve nonstationary models. Our solution method is based on combing a projection algorithm with backward induction. We exploit the fact that the costs of computers can be approximated as random but non-decreasing, which allows us the solve the problem of the agents backward, from a point in the far future. The period problem we solve by a projection method on an ergodic grid, which is known to be better suited when dealing with models with larger state space (Maliar and Maliar, 2015). To our knowledge, we are the first to solve for the transitional dynamics in a nonstationary model in the context of structural change.

Since the 1980s, the US labor market has undergone several significant changes. First, employment has shifted away from routine occupations since 1990. Routine occupations are middle-skill, repetitive jobs that follow explicit rules and are easily automated, such as clerks, accountants, and auditors. Non-routine occupations are jobs intensive in creativity and personal interactions at both ends of the skill distribution: high-skill cognitive jobs, such

as managers and engineers, and low-skill manual jobs, such as janitors and health aides.¹ Second, the growth rate of labor productivity increased from 1.6% before 1995 to 2.5% after 1995 (Jones, 2011). Third, the labor share of income declined by 7.5% between 1981 and 2007.²

Focusing on the cyclical dimension of these changes, the secular decline in routine jobs has been concentrated in recessions,³ and the ensuing recoveries have been jobless, i.e. employment recovers much slower than output, as can be seen in figure 1.⁴

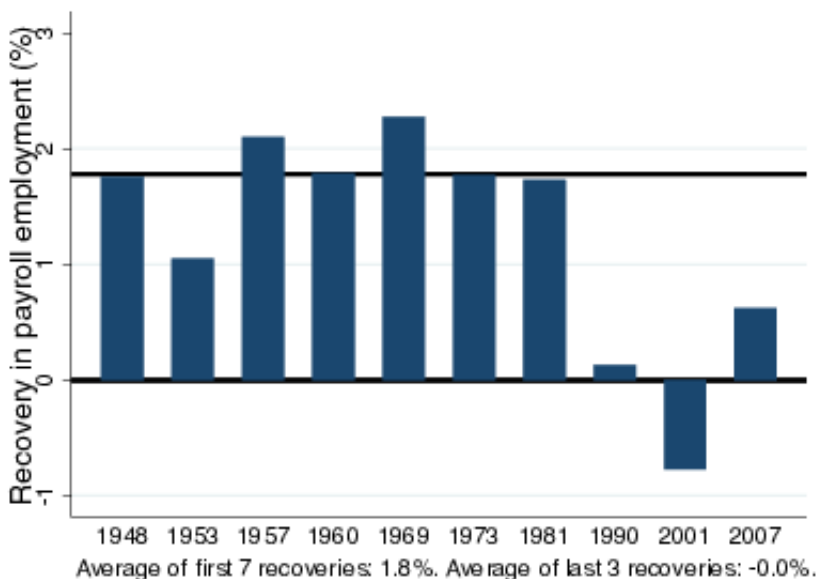


Figure 1: Payroll employment is slower to recover after the last three recessions, for a given recovery of output of 5%.

Source: Federal Reserve Economic Database (series PAYEMS and GDPC1, unfiltered). The recovery of employment is $\log(L_{\tau'}/L_{\tau})$, between the NBER trough τ and the time τ' such that output recovers by 5% ($\log(Y_{\tau'}/Y_{\tau}) = 5\%$), with linear interpolation.

At the same time, the cost of computers has been falling. Scholars disagree on the exact rate of decrease in the cost of computers,⁵ but agree that it was high—between 8% and

¹See Autor et al. (2006); Goos and Manning (2007), Goldin and Katz (2007), Autor and Dorn (2009) and Autor (2010).

²See Blanchard et al. (1997), and Rodriguez and Jayadev (2013).

³Jaimovich and Siu (2012b) find that 95% of the secular decline in routine jobs occurs in recessions.

⁴Figure 1 plots the recovery of employment for a given recovery of output of 5% and takes into account that recent recoveries have been slower. Not accounting for this different speed of output recovery would produce recoveries that are even more jobless. See also Gordon (1993); Andolfatto and MacDonald (2004); and Schreft et al. (2005).

⁵See Nordhaus (2007, Table 10, page 153) for a compilation of studies and methods.

27% per year.⁶ Figure 2, from the Bureau of Economic Analysis (BEA), illustrates this rapid decrease: between 1960 and 2010, the cost of computers declined at a rate of 18% per year. Computers are the only item in the BEA list whose price decreased—the overall price of equipment increased 1.3% per year. These differential price trends support the distinction between computer and non-computer capital. Indeed, several authors suggested

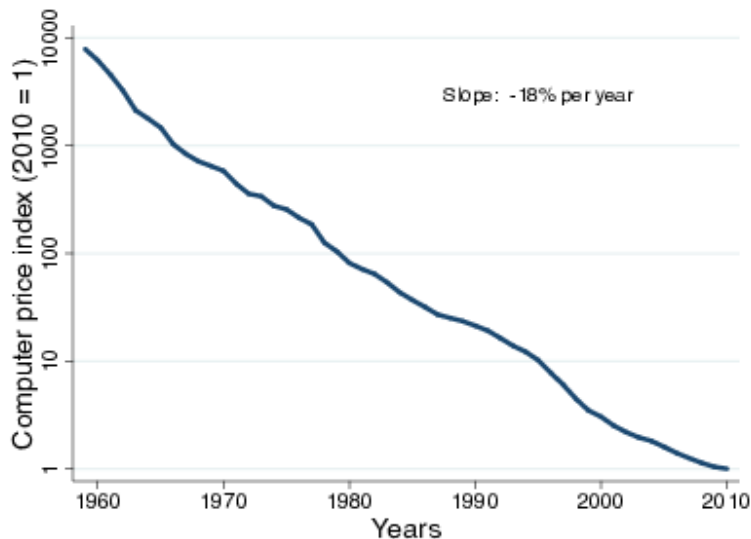


Figure 2: The cost of computers has an exponential decrease since 1960.

Source: Bureau of Economic Analysis, Price Indexes for Private Fixed Investment in Equipment by Type (Table 5.5.4U), line “Computers and peripheral equipment.”

computers (Autor et al., 2003, Oliner et al., 2007, Brynjolfsson and Hitt, 1994, Basu et al., 2001, Jorgenson, 2001, Karabarounis and Neiman, 2013, Saint-Paul and Bentolila, 2003), whose share of fixed investment accelerated in the 1980s (see Appendix D), as a plausible explanation for the changes we observe in the labor market.

This paper provides a theoretical contribution with a simple model of capital-labor substitution that reconciles the five facts—three medium-term changes over the last few decades and two short-term changes over the business cycle. This paper bridges the gap between growth and business cycles, between the literature on long-term technology adoption and the literature on the “cleansing effects” of recessions.⁷

The model starts from the neoclassical growth model and adds three main assumptions: first, computer capital substitutes routine labor services more than non-routine labor ser-

⁶See Sichel (1997, page 122), Nordhaus (2007, page 142), and Berndt and Rappaport (2001, page 271).

⁷See Helpman and Trajtenberg (1994); Bresnahan and Trajtenberg (1995); Caballero and Hammour (1994); Aghion and Saint-Paul (1998).

rices, second, the price of computer capital decreases over time, and third, recruiting labor services is costly. Shocks to Total Factor Productivity and household preferences give rise to economic fluctuations.

The paper shows that the first two assumptions of the model are sufficient to match the medium-term changes in the US economy. Firms producing adjust their input mix and substitute away from the expensive input of labor and into the cheaper input of capital. The lower demand for routine labor services shifts employment away from these occupations—an endogenous routinization of production. Employment reallocates into non-routine labor services with higher marginal productivity and this compositional effects rises the growth rate of labor productivity—an endogenous productivity speedup. Capital-labor substitution raises payments to capital at the expense of labor—an endogenous fall in the labor share of income.

The present model also clarifies why the price of computers has been falling since 1950 but starts affecting the labor market in the 1980s. Firms always adjust to the change in the price of computers, but the adjustment is small when the price is too high: computers were so expensive in the 1950s that firms produced output using routine labor services. Conversely, when computers are too cheap, firms have already replaced routine labor services and a further decrease in the price of computers is irrelevant for capital-labor substitution. The substitution of technology capital for routine labor services is quantitatively important when the price of the technology is in a specific range, a phase denoted as “technological upgrading.”

The model also clarifies that the substitutability between computer capital and routine labor services needs to be high enough in order to match the structural changes. A Cobb-Douglas production function has equal substitutability between all factors so the routine share of employment, the growth rate of labor productivity, and the labor share of income are all constant.

Combined with the third assumption of a hiring cost, the model also matches the cyclical changes of the US economy. Firms know that they will dismiss routine labor services in the medium-term during the technological upgrading phase. As computers complement non-routine labor services, firms also know that they will recruit more non-routine labor services. In a recession, forward-looking firms consider how to adjust the two types of labor services. If firms dismiss non-routine labor services, they will need to hire them back and pay a hiring cost. So firms avoid destroying non-routine labor services and hoard them during the recession. In contrast, dismissing routine labor services does not entail future hiring costs since their medium-term trend is declining. The burden of adjustment falls on routine labor

services, whose secular decline becomes concentrated in recessions.

Finally, the model can also account for jobless recoveries. As firms avoid dismissing non-routine labor services during recessions, they also refrain from recruiting them back temporarily, i.e. they “dishoard” non-routine labor services during the recovery. Firms also refrain from recruiting routine labor services because of their secular decline. Employment is stagnant even as output recovers, leading to a jobless recovery. In contrast, expensive computers earlier in time imply that routine labor services have a constant trend and that employment recovers to the pre-crisis level, leading to a “jobful” recovery.

A calibration of the model to fit the path of US GDP matches both the structural and the cyclical changes of the US labor market: the model matches the drop in the labor share of income in the data of 7.5% since 1981, the differential behavior of employment in routine and non-routine labor services during recessions, and the average recovery of employment of 0% in the last three recoveries in the US (for a given recovery of output of 5%).

The paper also tests the labor demand mechanism in this paper against the alternative labor supply mechanism of Jaimovich and Siu (2012b) where workers use the recession as an opportunity to quit their jobs and invest in education (more details in the literature review below). It uses the identification strategy of Andersen et al. (2012) with lightning flash density as an instrument for computer adoption: lightning strikes cause power surges, damage micro-computer chips, and predict computer adoption and labor productivity across US states after 1995. This paper tests whether unemployed workers in states that adopt computers were laid off (a labor demand mechanism) or quit their job (labor supply mechanism). Lightning strikes are a statistically and economically significant predictor of layoffs but not of quits among the unemployed. Moving from the 90% to the 10% percentile of lightning flashes raises the probability of having been laid off by 9 percentage points and suggests that the labor demand mechanism in this paper is more empirically plausible than Jaimovich and Siu.

Related literature. This paper relates to three strands of the literature: short-term adjustments of the labor market, the recent de-routinization and polarization of the labor market, and General Purpose Technologies.

On the short-term adjustments of the labor market, the closest paper is Jaimovich and Siu (2012b), who also use a distinction between routine and non-routine jobs to explain the concentration of routine job losses in recessions and jobless recoveries. They assume that the productivity of non-routine jobs increases exogenously faster than the productivity of routine jobs, so workers in routine jobs have an incentive to reallocate into non-routine jobs. Because of a period of retraining from routine to non-routine occupations, workers prefer to

reallocate when the opportunity cost is low, i.e. during recessions if wages are procyclical. Compared to the labor supply mechanism of Jaimovich and Siu, the model in this paper uses a labor demand mechanism with hiring costs for firms, which Section 5 finds to be more empirically plausible.

A second explanation for recent cyclical changes of the labor market is Berger (2012), who argues that the recent decrease in unionization allowed firms to fire unproductive workers more easily during the last three recessions. Berger also matches the emergence of longer jobless recoveries after the 1980s by distinguishing between two types of workers. A contribution of this paper is the emergence of jobless recoveries with a continuous mechanism (the progressive decline in the price of computers) rather than a structural break (the discrete switch from strong to weak unions).

Another contribution concerns the short-term predictions for other technologies and other time periods: the term “jobless recoveries” was invented by the New York Times in 1938 to describe the weak recovery from the Great Depression. Given that electricity was another General Purpose Technology adopted earlier in the century (David, 1990 and Field, 2011), this paper suggests that jobless recoveries are a recurrent issue in economic history, linked to the decrease in the cost of a new technology.

The second strand of the literature concerns the structural implications of the routinization and polarization of the labor market—i.e., when routine, middle-skill jobs lose importance in the employment structure and non-routine jobs at either end of the skill distribution gain employment share. Autor et al. (2008) documented the increase in upper-tail inequality since the 1990s and related it to a model of job polarization where computers complement high-education tasks and substitute middle-education tasks. Goos et al. (2014) explain the trend of polarization over 1993-2010 with routine-biased technical change and offshoring. Bárány and Siegel (2018) relate the trend of polarization to the shift away from manufacturing and into services. Relative to this literature, the contribution of this paper is two-fold: it explains why the price of computers has been falling since 1950 but its effects on the labor market are more pronounced since the 1980s; and it examines a single model that reconciles not only these structural changes but also the cyclical ones.

Third, the literature on General Purpose Technologies defined them with three characteristics: pervasive use in industry, decreasing cost for a given quality, and capacity to foster other innovations (Jovanovic and Rousseau, 2005, page 1185). If the General Purpose Technology is more substitutable to unskilled labor than to skilled labor, its adoption would increase the skill premium (Jovanovic and Rousseau, 2005, page 1205). This paper departs from the literature by studying the effects of the General Purpose Technology on the labor

share of income rather than on inequality.

2 A model of growth and business cycles

This section introduces a model to study the labor market consequences of computer adoption. The model uses computers for clarity but it can also apply to other General Purpose Technologies, such as electricity in the first half of the 20th century. Time is indexed as $t = 1, 2, \dots$

2.1 The household

A representative household consumes output, supplies labor, invests in capital, and rents the capital stock. It maximizes utility from consumption, net of disutility from labor supply:

$$\max \sum_{t=0}^{\infty} \theta^t \log \left(C_t - X_t \frac{\varepsilon}{1 + \varepsilon} L_t^{\frac{1+\varepsilon}{\varepsilon}} \right), \quad (2.1)$$

where θ is the discount factor, C_t is consumption, X_t is a labor supply shifter, ε is the Frisch elasticity of labor supply, and L_t is labor supply. The household has preferences as in Greenwood et al. (1988) with no income effects on labor supply.⁸ The labor supply shifter X_t has trend growth and serves *only* to ensure a balanced growth path with a constant trend of employment.

Capital is either computer capital $K_{C,t}$ or non-computer capital $K_{NC,t}$. The household accumulates capital with a perpetual inventory formula for each type of capital:

$$K_{C,t+1} = (1 - \delta_C) K_{C,t} + I_{C,t}, \quad (2.2)$$

$$K_{NC,t+1} = (1 - \delta_{NC}) K_{NC,t} + I_{NC,t}. \quad (2.3)$$

The household has access to a technology that transforms output into investment: one unit of output becomes one unit of non-computer investment $I_{NC,t}$ and one unit of output becomes e^{bt} units of computer investment $I_{C,t}$. Alternatively, the cost of non-computer investment is 1 and the cost of computer investment is e^{-bt} .

Considering consumption as the numeraire, the household has a budget constraint that

⁸See Jaimovich and Rebelo (2009) and Schmitt-Grohe and Uribe (2012) who find small income effects on labor supply in the short-term.

balances consumption and investment with labor income and capital income:

$$C_t + I_{NC,t} + \exp(-b_t) I_{C,t} = w_t L_t + r_{NC,t} K_{NC,t} + r_{C,t} K_{C,t} + \text{profits}_t, \quad (2.4)$$

where w_t is the wage, $r_{J,t}$ are the rental rates of capital ($J = I, N$), and profits_t are the firm's profits in period t , which the household takes as given.

The first crucial assumption is the medium-term increase in the productivity b_t :

Assumption 1. *The logarithm b_t of the productivity of the computer-producing technology increases exogenously with time: $b_t \nearrow$ in t .*

Alternatively, the cost of computers e^{-b_t} decreases with time.

2.2 The firm

The production function uses four inputs: two types of capital (computer capital $K_{C,t}$ and non-computer capital $K_{NC,t}$) and two types of labor (routine labor services $L_{R,t}$ and nonroutine labor services $L_{NR,t}$). The production function is:

$$Y_t = A_t K_{NC,t}^\alpha L_{NR,t}^\beta M_t^\gamma, \quad M_t = \left(K_{C,t}^{\frac{\sigma-1}{\sigma}} + L_{R,t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad (2.5)$$

where A_t is Total Factor Productivity (TFP) and represents fluctuations driven by technology. The production function has constant returns to scale, with $\alpha + \beta + \gamma = 1$. This production function has Cobb-Douglas aggregation of three factors: non-computer capital $K_{NC,t}$, nonroutine labor services $L_{NR,t}$, and a third factor M_t , which is a Constant-Elasticity-of-Substitution aggregation between computer capital $K_{C,t}$ and routine labor services $L_{R,t}$.

Krusell et al. (2000) use this production function to explain the increase in income inequality with capital-skill complementarity, whereby an increase in capital investment contributes to increasing the skill premium by increasing the marginal product of skilled labor faster than that of unskilled labor. Autor and Dorn (2009, page 11) also use this function to explain the recent disappearance of middle-skill, routine occupations: as firms invest more in computer capital, they increase employment of middle-skill routine labor services slower than low-skill or high-skill nonroutine labor services.

The second crucial assumption is the gross substitutability of computer capital and routine labor services:

Assumption 2. *The elasticity of substitution between computer capital and routine labor services is at least greater than 1: $\sigma \geq 1$.*

Autor et al. (2003) find that computer investment is correlated with a decrease in routine labor services and an increase in nonroutine labor services. The case $\sigma > 1$ captures that difference: the elasticity of substitution between routine labor services and computers is greater than the elasticity of substitution between nonroutine labor services and the Constant-Elasticity-of-Substitution aggregate of computers and routine labor services.⁹ Intuitively, a computer can more easily replace automated occupations, such as bank tellers or cashiers, than nonroutine occupations, such as managers and engineers. The case $\sigma = 1$ represents the Cobb-Douglas benchmark.

A representative firm demands labor and capital and produces output. It operates under perfect competition and has profits

$$\text{profits}_t = Y_t - w_t(L_{NR,t} + L_{R,t}) - \sum_{I=NC,C} r_{I,t}K_{I,t} - \sum_{J=NR,R} c_J(L_{J,t+1} - L_{J,t})^+, \quad (2.6)$$

where $c_J, J = NR, R$ is the unit cost of recruiting nonroutine or routine labor services and $x^+ = \max(x, 0)$ is the positive operator. Linear adjustment costs to labor are common in the literature (Bentolila and Saint-Paul, 1994) as opposed to quadratic adjustment costs for capital (Caballero and Hammour, 1994). The firm reverts profits to the household and uses the household's discount factor weighted by marginal utility from consumption to compute the present discounted value of profits (see Appendix A).

The third crucial assumption bears on the adjustment cost:

Assumption 3. *The costs of hiring are non-negative: $c_{NR} \geq 0$, and $c_R \geq 0$.*

Hiring costs capture the firm-specific value of a match, such as a training cost paid by the firm for a new worker. The extensive literature on hiring costs supports this assumption: Blatter et al. (2012) estimate hiring costs around one quarter of wages using a dataset of Swiss firms, which Del Boca and Rota (1998) confirm using a survey of Italian firms. Hamermesh (1993) reports similar values for the United States: in 1980, the average employer spent 42 hours and two quarters of wages recruiting and training a new hire.¹⁰

⁹This assumption is both a relative statement, with computers being more substitutable to routine labor services than to nonroutine labor services, and an absolute statement, with the elasticity of substitution between routine labor services and computers being greater than 1.

¹⁰Assumption 3 implies that hiring costs are larger than firing costs, which is consistent with Hamermesh: "The [1965] study found separation costs to be much smaller." For simplicity, the model assumes that firing costs are zero.

2.3 Environment

There are two exogenous independent variables, aggregate productivity and relative costs of computers. Aggregate productivity follows a standard AR(1) in logs process

$$\log A_{t+1} = \rho \log A_t + \eta_{t+1} \quad (2.7)$$

with $\rho = 0.95$ and $\eta \sim N(0, 0.009^2)$.

The relative price of computes has been falling at an exponential rate. This is captured in variable b_t (see equation (2.4)). We model this process by a stochastic non-decreasing series.

Assumption 4. *The price of computers starts at $\exp(-b_0) = \exp(-\underline{b})$ follows the following process*

$$b_{t+1} = \begin{cases} b_t, & \text{with probability } P \\ b_t + \kappa, & \text{otherwise} \end{cases} \quad (2.8)$$

until it reaches $b_t < \bar{b}$.

The cost of computers b_t can take only values from $\{\underline{b}, \underline{b} + \kappa, \underline{b} + 2\kappa \dots, \bar{b}\}$ (which has N elements and $N\kappa = \underline{b} - \bar{b}$) and $b^j = \underline{b} + (j - 1)\kappa$. In the process of transition it takes all of them one by one, but it does not have to change every period (so it might take more than N periods to reach \bar{b}). The fact that this process is de facto a one way street, makes it possible to solve the model in a backward fashion.

2.4 Equilibrium

Labor market clearing requires that labor supply equal labor demand: $L_t = L_{NR,t} + L_{R,t}$. This condition, in combination with the utility function, implies that labor supply is perfectly substitutable between routine and nonroutine labor services. This unrealistic assumption allows the paper to examine the contribution of labor demand alone in explaining changes of the US labor market, as opposed to the labor supply mechanism of Jaimovich and Siu.¹¹

¹¹With costly reallocation between the two types of labor services, the routine wage is lower than the nonroutine wage and workers providing routine labor services remain competitive for a longer period of time, which would attenuate the medium-term effects of the model, but would strengthen the short-term effects of the model by a mechanism similar to Jaimovich and Siu, since the wage is the opportunity cost of reallocation and workers prefer to switch jobs during a recession.

The clearing of the product market follows from the budget constraint, the definition of the firm's profits, and the clearing of the labor market. The clearing of the capital market is implicit in the use of a single symbol for capital supply and capital demand.

An equilibrium of this economy is a set of quantities (consumption C_t , investments $I_{C,t}$ and $I_{NC,t}$, capital stocks $K_{C,t}$ and $K_{NC,t}$, employment quantities L_t , $L_{NR,t}$ and $L_{R,t}$, and output Y_t) and prices (rental rates $r_{C,t}$ and $r_{NC,t}$, and wages w_t), conditional on exogenous variables (TFP A_t , the productivity b_t of the computer-producing technology, and the labor supply shifter X_t), such that the household maximizes utility (2.1) subject to the capital accumulation constraints (2.2-2.3) and the budget constraint (2.4); the firm maximizes the present discounted value of profits (2.6) subject to the production function (2.5); and all markets clear. This model nests the Ramsey growth model, which corresponds to a two-factor production function ($\gamma = 0$), no adjustment costs ($c_{NR} = c_R = 0$), and constant labor supply.

The full characterization of the model is in Appendix A. This appendix proves existence and uniqueness of the equilibrium and of a balanced growth path. The model is analytically intractable and has no closed-form solution. The next section examines a simplified version of the model to examine the structural changes of the labor market and Section 4 uses the general version of the model to examine the cyclical changes.

2.5 Solution method

This section describes the solution methods. The present version of the paper includes results from the previous solution method, which was perfect foresight deterministic version. The full stochastic results are still work in progress.

The state space consists of six variables: $\Sigma_t \equiv [A_t, K_{C,t}, K_{NC,t}, L_{R,t}, L_{NR,t}]$ and b_t .

2.5.1 Terminal state

Here, $b_t = b_{t+1} = \bar{b}$ forever, so the solution becomes a standard infinite horizon recursive model and b becomes a standard parameter in the budget constraint. Such model can be solved using standard methods.

1. Discretise Σ .
2. Make a guess for prices ($r_C(\Sigma)$, $r_{NC}(\Sigma)$, $w(\Sigma)$) and aggregate laws of motion ($K'_C(\Sigma)$, $K'_{NC}(\Sigma)$, L'_R , L'_{NR} and $m'(\Sigma)$). Using these, the agents make forecast about prices in the future.

3. Solve for the policy rules of the firms and households, described by the following Belmann equations

$$V^F(\Sigma_t) = \max_{L_{R,t}, L_{NR,t}} \Pi_t + \mathbf{E} [m_{t,t+1} V^F(\Sigma_{t+1})],$$

$$V^H(\Sigma_t) = \max_{L_{R,t} + L_{NR,t}, I_C, I_{NC}} u(C, L^S) + \beta \mathbf{E} [V^H(\Sigma_{t+1})].$$

4. Check for market clearing and adjust prices
5. Iterate 3 and 4 until convergence.

2.5.2 Transition states

Once the model solution for the terminal state is known, we can proceed to solve for the transitional dynamics backwards. Assume we already know the solution for the model when $b = b^{j+1}$ and we are solving for the solution for $b = b^j$.

1. Guess all the policy function for all the transitional states
2. Simulate the whole transition path
3. Solve the model backwards, starting from the last but one state of b (that is b^{N-1}) back to b^1
 - (a) Discretise the state space Σ using the method described in Maliar and Maliar (2015)
 - (b) Make a guess for prices ($r_C(\Sigma, b^j)$, $r_{NC}(\Sigma, b^j)$, $w(\Sigma, b^j)$) and aggregate laws of motion ($K'_C(\Sigma, b^j)$, $K'_{NC}(\Sigma, b^j)$, $L'_R(\Sigma, b^j)$, $L'_{NR}(\Sigma, b^j)$, $m'(\Sigma, b^j)$, *today*)
 - (c) Solve for the policy rules of the firms and households,

$$V^F(\Sigma_t, b^j) = \max_{L_{R,t}, L_{NR,t}} \pi_t + P \mathbf{E} [m(\Sigma_{t+1}, b^j) V^F(\Sigma_{t+1}, b^j)]$$

$$+ (1 - P) \mathbf{E} [m(\Sigma_{t+1}, b^{j+1}) V^F(\Sigma_{t+1}, b^{j+1})] \quad (2.9)$$

$$V^H(\Sigma_t, b^j) = \max_{L_{R,t} + L_{NR,t}, I_C, I_{NC}} u(C, L^S) + P \beta \mathbf{E} [V^H(\Sigma_{t+1}, b^j)]$$

$$+ (1 - P) \beta \mathbf{E} [V^H(\Sigma_{t+1}, b^{j+1})] \quad (2.10)$$

while exploiting the fact that $r_C(\Sigma, b^{j+1})$, $r_{NC}(\Sigma, b^{j+1})$, $w(\Sigma, b^{j+1})$, $K'_C(\Sigma, b^{j+1})$, $K'_{NC}(\Sigma, b^{j+1})$, $L'_R(\Sigma, b^{j+1})$, $L'_{NR}(\Sigma, b^{j+1})$ and $m'(\Sigma, b^{j+1})$ are already known.

- (d) Check market clearing and adjust prices
 - (e) Iterate 3 and 4 until convergence
4. Iterate 2 and 3 until convergence

3 Medium-term trends

This section simplifies the model as a first step to understand under which conditions capital-labor substitution leads to the routinization of production, to a productivity speed-up, and to a decline in the labor share of income.

Two simplifications render the model analytically tractable. First, hiring costs are zero, with $c_{NR} = c_R = 0$, so the firm is free to adjust labor. Second, capital accumulates immediately and depreciates fully after one period: $K_{NC,t} = I_{NC,t}$ and $K_{C,t} = I_{C,t}$. The firm has no frictions and makes zero profits in all periods. Then the budget constraint of the household becomes $C_t = w_t L_t$.¹² The household cannot smooth consumption and the intertemporal utility maximization is equivalent to a set of independent maximization programs, one for every period.

3.1 Endogenous structural changes

This subsection describes how Assumptions 1 and 2, with the restriction $\sigma > 1$ and the above simplifications, match the three structural changes of the labor market, but the Cobb-Douglas case with $\sigma = 1$ cannot. In addition, this subsection excludes TFP shocks ($A_t = A$). The time-varying exogenous variables are the labor supply shifter X_t and the productivity b_t of the computer-producing technology.

Full depreciation of capital pins down the rental rates of capital as the prices of investment. The missing price in the economy is the wage, which follows from the factor price frontier in the next lemma. (Appendix C details all proofs in this subsection.)

For $\sigma > 1$, the wage is the unique solution to the factor price frontier:

$$1 = \frac{1}{A_t} \left(\frac{1}{\alpha}\right)^\alpha \left(\frac{w_t}{\beta}\right)^\beta \left(\frac{(r_{C,t}^{1-\sigma} + w_t^{1-\sigma})^{\frac{1}{1-\sigma}}}{\gamma}\right)^\gamma. \quad (3.1)$$

¹²This equation follows the budget constraint, from the assumption that capital equals investment, and the equilibrium in the capital markets, where the household sells capital to the firm at marginal cost with $r_{NC,t} = 1$ and $r_{C,t} = \exp(-b_t)$.

The left-hand side of the factor price frontier is the marginal benefit of selling one more unit of output, whose price is normalized to 1. The right-hand side is the marginal cost: the inverse of Total Factor Productivity multiplied by the marginal price of each Cobb-Douglas factor divided by its share and raised to that share. The marginal price of non-computer investment is 1, the marginal price of nonroutine labor services is the wage w_t , and the marginal price of the third factor is a Constant-Elasticity-of-Substitution relationship between the rental rate of computer capital and the wage.

The aggregation between the rental rate of computers and the wage is the key to the model's ability to match the structural changes. Consider the two limiting scenarios of expensive and cheap computers. When computers are expensive, the term $r_{C,t}^{1-\sigma}$ vanishes from the equation and the factor price frontier is close to that of a labor-intensive production function with three Cobb-Douglas factors of non-computer capital, nonroutine labor services, and routine labor services:

$$\lim_{b_t \rightarrow -\infty} Y_t = A_t K_{NC,t}^\alpha L_{NR,t}^\beta L_{R,t}^\gamma.$$

When computers are cheap, the term $r_{C,t}^{1-\sigma}$ gains importance, the term $w_t^{1-\sigma}$ vanishes from the equation, and the factor price frontier is close to that of a capital-intensive production function with three Cobb-Douglas factors of in non-computer capital, nonroutine labor services, and computer capital:

$$\lim_{b_t \rightarrow \infty} Y_t = A_t K_{NC,t}^\alpha L_{NR,t}^\beta K_{C,t}^\gamma.$$

The transition between these two limiting production functions is a phase of “technological upgrading” that matches the structural changes of the US labor market. Employment shifts away from routine labor services, which have a share of γ in the labor-intensive production function and a share of 0 in the capital-intensive production function. Computers do not contribute to output and labor productivity in the labor-intensive production function but they do contribute in the capital-intensive production function, so labor productivity speeds up. The labor share of income decreases from $\beta + \gamma$ in the labor-intensive production function to β in the capital-intensive production function.

The rest of this section shows these intuitive results analytically. The next proposition shows that a decrease in the cost of computers causes a decrease in the routine share of employment.

Proposition 5. For $\sigma > 1$, the routine share of employment decreases:

$$\lim_{b_t \rightarrow -\infty} \frac{L_{R,t}}{L_t} = \frac{\gamma}{\beta + \gamma}, \quad \lim_{b_t \rightarrow \infty} \frac{L_{R,t}}{L_t} = 0.$$

Moreover, the productivity b_t of the computer-producing sector impacts the logarithm of the routine share of employment, $s_t = \log(L_{R,t}/L_t)$, with increasing importance:

$$\lim_{b_t \rightarrow -\infty} \frac{\partial s_t}{\partial b_t} = 0, \quad \lim_{b_t \rightarrow \infty} \frac{\partial s_t}{\partial b_t} = (1 - \sigma) \left(1 + \frac{\gamma}{\beta}\right).$$

The next proposition shows that progress in the computer-producing technology causes a productivity speedup in the wider economy.

Proposition 6. For $\sigma > 1$, b_t impacts labor productivity $\pi_t \equiv \log Y_t/L_t$ with increasing importance:

$$\lim_{b_t \rightarrow -\infty} \frac{\partial \pi_t}{\partial b_t} = 0, \quad \lim_{b_t \rightarrow \infty} \frac{\partial \pi_t}{\partial b_t} = \frac{\gamma}{\beta}.$$

The next proposition shows that a decrease in the price of computers causes a decrease in the labor share of income.

Proposition 7. For $\sigma > 1$, the labor share of income decreases from $\beta + \gamma$ to β , linked to the relative price of computer capital:

$$\frac{w_t L_t}{Y_t} = \beta + \gamma \left(1 + \left(\frac{r_{C,t}}{w_t}\right)^{1-\sigma}\right)^{-1} \searrow \text{ in } t, \quad \lim_{b_t \rightarrow -\infty} \frac{w_t L_t}{Y_t} = \beta + \gamma, \quad \lim_{b_t \rightarrow \infty} \frac{w_t L_t}{Y_t} = \beta.$$

The next corollary shows that $\sigma > 1$ is necessary to match the structural changes. In the benchmark case of $\sigma = 1$, the routine share of employment, productivity growth, and the labor share of income are all constant due to the Cobb-Douglas structure.

Corollary 8. If $\sigma \rightarrow 1$, the effect of computers on labor productivity, the labor share of income, and the routine share of employment are independent of computer productivity:

$$\frac{\partial s_t}{\partial b_t} \Big|_{\sigma \rightarrow 1} = \frac{\partial^2 \pi_t}{\partial b_t^2} \Big|_{\sigma \rightarrow 1} = \frac{\partial \log(w_t L_t / Y_t)}{\partial b_t} \Big|_{\sigma \rightarrow 1} = 0.$$

3.2 Illustration

To illustrate the mechanism numerically, this subsection calibrates the simplified model with full capital depreciation and no hiring costs. It specifies the two crucial parameters (the path

of b_t and the elasticity of substitution σ), with the remaining parameters calibrated in the full model in Section 4. The cost of computers decreases at rate $\phi = 18\%$ per year (from the Bureau of Economic Analysis). The value of σ relates to a substantial literature on the estimation of the elasticity of substitution between aggregate capital and aggregate labor. Using cross-country variation in the price of investment, Karabarbounis and Neiman (2013) estimate the elasticity of substitution at 1.25. Accounting for technological change that may be biased toward some factors, Antràs (2004) estimates elasticities of substitution that are not statistically different from 1. Krusell et al. (2000) disaggregate capital and labor and estimate the elasticity of substitution between unskilled labor and equipment at 1.67 from time-series US data. Since the form of their production function is closest to this paper and their estimation is from contemporary US data, this paper uses $\sigma = 1.67$.¹³

Figure 3 shows the behavior of the economy in the medium-term with fictional dates. Total Factor Productivity is constant, with $A_t = A$. The labor supply shifter X_t grows and exactly offsets the increase in the wage so the economy has constant employment (as in the balanced growth path of Appendix B).

This example illustrates the labor-intensive phase that lasts roughly until the 1980s: computers are too expensive, the firm relies on routine labor services, the share of computer capital in total capital is near zero. The routine share of employment, the growth rate of labor productivity, and the labor share of income are all roughly constant.

This example also illustrates the technological upgrading phase that starts in the 1980s. The firm starts replacing routine labor services with cheaper computers, employment reallocates into nonroutine labor services L_{NR} with higher marginal productivity, and payments to capital increase at the expense of routine labor.

This transition also explains why the price of computers has been falling since 1950 but starts affecting the labor market three decades later in the 1980s. Firms' adjustment to the changes in the price of computers is quantitatively small in the labor-intensive phase and becomes quantitatively important in the technological upgrading phase. Note that this transition is continuous and has no threshold effects.

This illustration concludes the simplified model and the implications of Assumptions 1

¹³ Further support for this value comes from the calibration of the general case of the model: Figure 11 in the Appendix shows that the model predicts a decline in the labor share of income that is similar to that in the data.

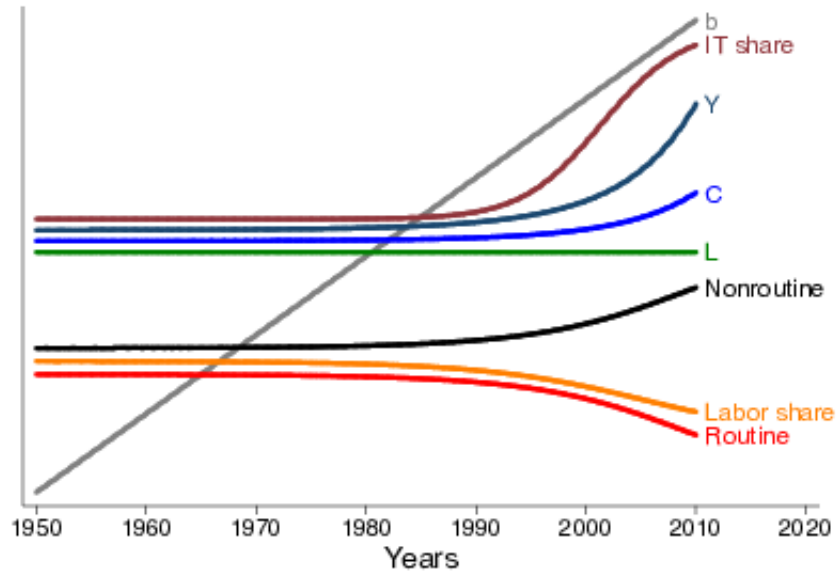


Figure 3: The special case of the model matches the medium-term changes: a linear increase in the productivity of the computer-producing sector causes a decline in the routine share of employment, a speedup in labor productivity, and a fall in the labor share of income.

and 2 for the structural changes of the labor market. The next section examines the general version of the model and its short-term implications, given that the literature on the “cleaning effects of recessions” suggests that downturns are special times for restructuring production (Caballero and Hammour, 1994 and Aghion and Saint-Paul, 1998).

4 Short-term predictions

This section considers the general version of the model, with positive hiring costs and accumulation of capital. Compared to the special case of the model above, the main difference is the firm’s choice of the optimal time to dismiss labor services. Firms know that nonroutine labor services are expanding in the medium-term. Instead of dismissing nonroutine labor services during recessions and paying a hiring cost in the recovery, firms hoard or retain nonroutine labor services during the recession. In contrast, routine labor services are declining and do not imply hiring costs in the recovery. The burden of adjustment falls on routine labor services. The interaction between short-term hiring costs—a cyclical characteristic—and the secular decline in the price of computers—a structural characteristic—implies that routine

job losses are concentrated in recessions during the technological upgrading phase.¹⁴ This interaction explains why, if the price of computers has been falling since 1950, it is only in recent decades that we see its effects on the cyclical behavior of the labor market with the acceleration of the decline in routine jobs in recessions and jobless recoveries.

4.1 Calibration

The calibration of the model uses the same values for the elasticity σ and the rate of decrease ϕ as Section 3.2. The hiring costs are between zero and one quarter of wages initial wages w_{1947} .¹⁵ The share of non-computer capital is $\alpha = 0.3$, the standard share of capital in aggregate income. The nonroutine share $\beta = 0.39$ of aggregate output is from the Current Population Survey in 2007, identifying workers as nonroutine if they are below the median of an index of routinization defined in subsection 4.3.¹⁶ The quarterly discount factor is $\theta = 0.99$. The elasticity of labor supply is $\varepsilon = 1$, consistent with Keane (2011, page 1042). The depreciation of non-computer capital is $\delta_{NC} = 1.5\%$ and the depreciation of computer capital is $\delta_C = 7.5\%$ (6% and 30% in annual terms).¹⁷ Henceforth, the model considers only TFP shocks. The labor supply shifter has no cyclicality and grows at a rate that ensures a constant trend in employment.

Parameter	α	β	γ	σ	c_{NR}, c_R	θ	ε	δ_C	δ_{NC}
Value	0.3	0.39	0.31	1.67	0, 0.1, or 0.8	0.99	1	7.5%	1.5%

Table 1: Parameter values for the calibration of the model.

4.2 Acceleration of routinization in simulations

Since the model is analytically intractable, this subsection uses numerical simulations to illustrate the following property: even with symmetric hiring costs, with $c_{NR} = c_R = c$, routine labor services are more responsive to a recession than nonroutine labor services.

¹⁴Nevertheless, investment in computer equipment is procyclical, as it is in the data (see Appendix D for details).

¹⁵This value is consistent with previous literature: the adjustment costs in Berger (2012, page 23) are 7 months of wages. In the calibration with US GDP, spending on hiring costs is at most 0.2% of GDP.

¹⁶I compute $\beta = 0.392$ in two steps: I use the Current Population Survey to estimate the share of routine labor income among all labor income of 56 percent in 2007, which I then multiply by the share of labor income in value added of 70 percent.

¹⁷See Karabarbounis and Neiman (2014) for the implications of the fall in the price of equipment on depreciation and the labor share of income.

Each simulation is denoted $i \in \{1..300\}$ and has a path for TFP A_t^i that follows the standard AR(1) process in Kydland and Prescott (1982): $\log A_t^i = 0.95 \log A_{t-1}^i + 0.009 \times \mathcal{N}(0, 1)$. These simulations use a path for TFP with no trend growth. (Note that this calibration of TFP shocks concerns only these simulations, while the fit of US data in the next subsection computes the implied TFP shocks directly from the data.) The simulations solve two models, without and with adjustment costs ($c = 0$ or $c = 0.1$), whose solutions are denoted $\{L_{NR,c,t}^i, L_{R,c,t}^i\}$. The elasticity of employment with respect to negative TFP shocks is the coefficient of a regression of $\Delta \log L_{J,c,t}$ on $\Delta \log A_t$, for $\Delta \log A_t < 0$, $J = NR, R$, and $c = 0, 0.1$.

The simulations confirm that the burden of adjustment of a TFP shock falls on routine labor services more than on nonroutine labor services. The elasticity of employment with respect to TFP shocks in the technological upgrading phase is similar without adjustment costs: 0.71 for routine labor services and 0.72 for nonroutine labor services. With symmetric adjustment costs, the elasticity decreases for both types of services, but is much higher for routine labor services at 0.31 compared to 0.05 for nonroutine labor services, or six times higher.¹⁸¹⁹

4.3 Acceleration of routinization in a fit to US GDP

An alternative to the numerical simulations is to fit the model with US GDP. The adjustment costs are now $c_{NR} = 0.8$, which correspond to one quarter of wages at the beginning of the period, and $c_R = 0$. The simplifying assumption of zero routine hiring costs implies that no cyclical force holds back the hiring of routine occupations in the recovery.²⁰

The growth in the labor supply shifter X_t requires delicate attention. Recent recoveries are not only jobless but also slow (Galí et al., 2012): output recovers faster after early recessions than recent ones. For a given recovery in output, recent recoveries last longer. A constant growth in the disutility of labor supply would imply that the household is less willing to work in recent recoveries than in earlier ones, which would bias in favor of jobless recoveries. To remove this labor supply mechanism and decrease the chances of matching

¹⁸Conversely, for positive technology shocks, it is nonroutine labor services that are more responsive at an elasticity of 0.25 compared to 0.05 for routine labor services.

¹⁹Subsection 4.4 uses positive hiring costs for the whole period of 1947-2010 and clarifies that recent recessions are followed by jobless recoveries because the economy enters the technological upgrading phase with cheaper computers, not because hiring costs are higher.

²⁰Zero routine hiring costs also imply that aggregate employment responds to a recession, whereas routine hiring costs with one quarter of wages would prevent dismissing routine labor services during early recessions and imply a-cyclical employment. These values are also consistent with Hamermesh, who reports “an average hiring cost for all occupations of \$910, but an average for managerial and professional workers of \$4,660.”

jobless recoveries, the calibration specifies a growth rate for X_t of 3.8% before 1985, larger than the growth rate of 1.71% after 1985. Employment has no trend over the whole period.

The numerical solution computes the shocks to TFP A_t that match US GDP exactly. Specifically, the characterization of the equilibrium in Appendix A gives n equations with $n+1$ unknowns for each time period, the extra unknown being the TFP shock. The numerical solution pins down the model by using output as an additional series, which obtains n equations in n unknowns. This approach matches output by construction and computes the TFP shocks that are exactly consistent with output. It also avoids computing a nested fixed point and allows an efficient calibration that solves in a few seconds.²¹

This calibration is similar to the growth accounting exercise of imputing the “Solow residuals” as unobserved TFP shocks: the model and the regression fit the path of output perfectly by computing the implied TFP shocks. This calibration is not a test of the model, since the path of output is taken from the data, but illustrates the mechanism of nonroutine hoarding during recent recessions in perfect foresight. Berger (2012) uses a similar approach and computes the path of aggregate-level TFP shocks that are exactly consistent with output during the 2007 recession.

The key mechanism in the model is the differential behavior of routine and nonroutine labor services in recessions. Figure 4 illustrates this difference in the calibration of the model over the last four decades. The firm hoards nonroutine occupations during recessions, rather than firing them in a recession and hiring them again in a recovery. In recent decades, a recession accelerates the secular decrease in routine labor services, which do not recover back to peak.²²

To take this prediction to the data, I use the Current Population Survey matched to the Occupational Information Network. For a measure of routinization, Autor et al. (2003) classify routine jobs as high in automation, low in personal interactions, and low in creativity. An index of routinization combines these three measures:

$$\text{routinization}_j = \text{automation}_j - \text{assisting others}_j - \text{level of creativity}_j,$$

where j indexes occupations. I aggregate employment into employment quartiles by routinization index for each peak year, divide employment by working-age population,²³ and

²¹See Conlon (2010) for using the AMPL software to solve constrained-optimization problems.

²²Figure 4 does not plot the recovery to facilitate comparison with Figure 5, which has breaks in the Standard Occupational Classification in 1982, 1992, and 2003. Figure 6 details the behavior of total employment in the recovery taking the differential speed of recovery into account.

²³I use the series USAWFPNA from the Federal Reserve Economic Database.

normalize the employment share quartiles at 100 in the peak year.

Figure 5 is the empirical counterpart of Figure 4 and plots the time-series of each quartile by decade.²⁴ The least routinizable occupations, in the first quartile, represent nonroutine and expanding jobs: they have the largest medium-term increase in all decades and never decrease during recessions. Occupations that are neither routine nor nonroutine, in the second quartile, represent cyclical jobs: they increase during expansions and decrease during recessions. The most routinizable occupations, in the third and fourth quartiles, represent declining jobs, intensive in automation and with little scope for personal interactions or creativity. Employment in these occupations follows a step function: flat or declining in the 1990 and 2000 expansions and decreasing during recessions. Between 2007 and 2010, employment in upper quartiles of routinization decreased by 5.8 million jobs, around 80% of job losses over the period, a figure that is similar for the 1990 and 2001 recessions.

4.4 Jobless recoveries

The calibration of the model to fit US GDP also matches jobless recoveries. The intuition for the mechanism comes from Figure 4: because the firm hoards nonroutine labor services during a recession, it “dishoards” them during the recovery. That is, the firm refrains from recruiting nonroutine labor services until the labor demand that would prevail without hiring costs (which is the dotted line or the lower end of the inaction band in Figure 4) increases and meets the current stock of nonroutine labor services. Routine labor services adjust freely: they return to peak in early recoveries and to the declining trend in late recoveries. The trajectory of routine labor services is V-shaped in early recessions and L-shaped in late recessions. The implication is that after early recessions, the firm dishoards nonroutine labor services and recruits routine labor services back to peak, leading to a “jobful” recovery. After late recessions, the firm also dishoards nonroutine labor services but routine labor services return to their declining trend, so aggregate employment is stagnant even as output recovers, leading to a “jobless” recovery.

Figure 6 shows the recovery of employment in two numerical exercises. The first exercise solves a model where computers remain expensive and the productivity b_t of the computer-producing sector is constant at the 1947 level. The second exercise solves a model where the price of computers falls at rate $\phi = 18\%$. When computers remain expensive, the average recovery of employment is the same for all recessions and around 0.7%. When

²⁴The occupational classification of the CPS changed every decade. The 2003-2010 panel uses the 6-digit Standard Occupation Classification of 2000 (SOC2000). The remaining panels use the OCC1990 variable provided by IPUMS.

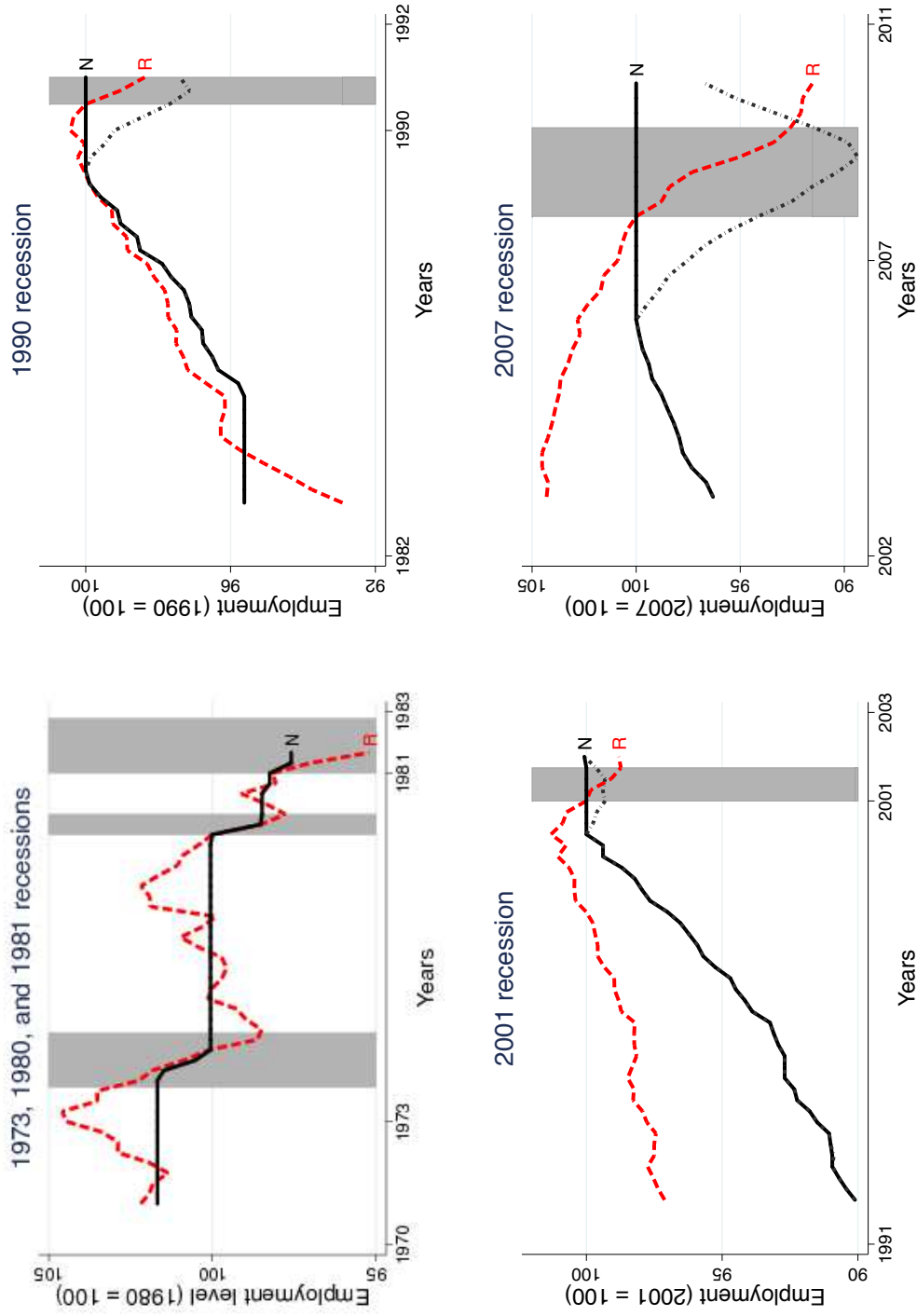


Figure 4: The calibration of the model predicts an acceleration of the decline in routine occupations during recent recessions. Shaded areas are NBER recessions. Nonroutine labor services “N” are the solid black line, routine labor services “R” are the dashed red line. The dot-dashed line is the lower end of the inaction band for nonroutine labor services, i.e. the region where the firm has no incentive to adjust nonroutine labor services because the gains from reducing labor demand are lower than the hiring costs incurred later. This lower inaction band is far below employment in nonroutine labor services before 1984 and is omitted for clarity.

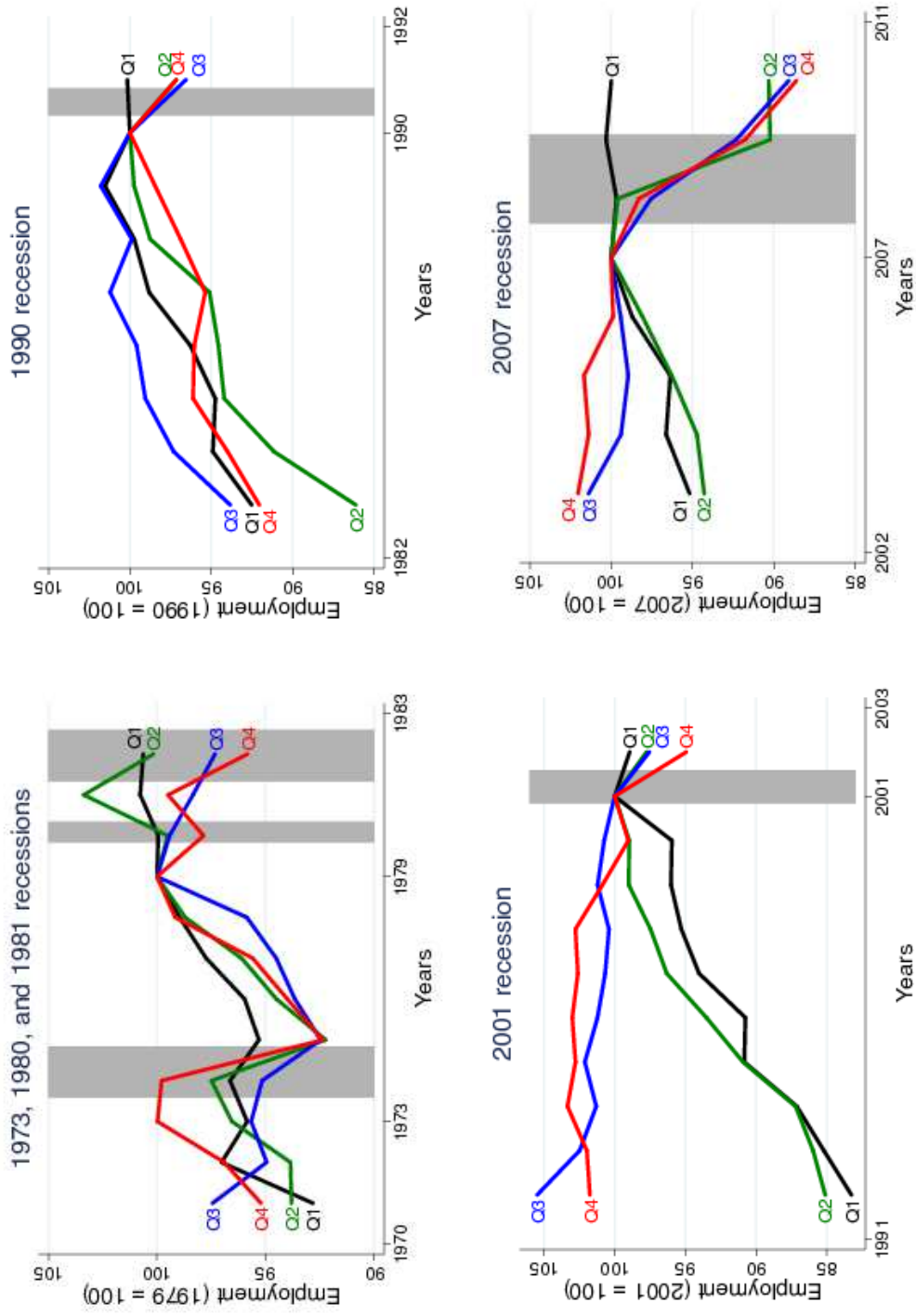


Figure 5: The Current Population Survey also displays an acceleration of the decline in routine occupations during recent recessions.

Details: "Q#" is quartile #, (see text for the definition of employment quartiles). Source: Current Population Survey and Occupational Information Network. Shaded areas are NBER recessions.

computers become cheaper, the recovery of employment drops from an average of 0.71% for early recessions (1948 to 1981) to an average of -0.04% for late recessions (1990 to 2007). The model explains jobless recoveries in the sense that cheaper computers predict a weaker recovery of employment after the last three recessions while expensive computers predict a similar recovery for employment across all recessions.

The model over-predicts the joblessness of the recovery after the 2007 recession compared to the data in Figure 1. One possible explanation is credit market disruptions caused firms to lay off more workers (Chodorow-Reich, 2014). With the end of the financial crisis, firms may have used their credit access to hire back laid off workers, a mechanism that is absent from the model. Another possible explanation is that the model pools together nonroutine labor services at the top and bottom of the skill distribution and both types of labor services are hoarded during the recession. In reality, nonroutine labor services at the bottom of the distribution may be fired during the recession and hired back in the recovery, causing the model to understate the recovery of employment.

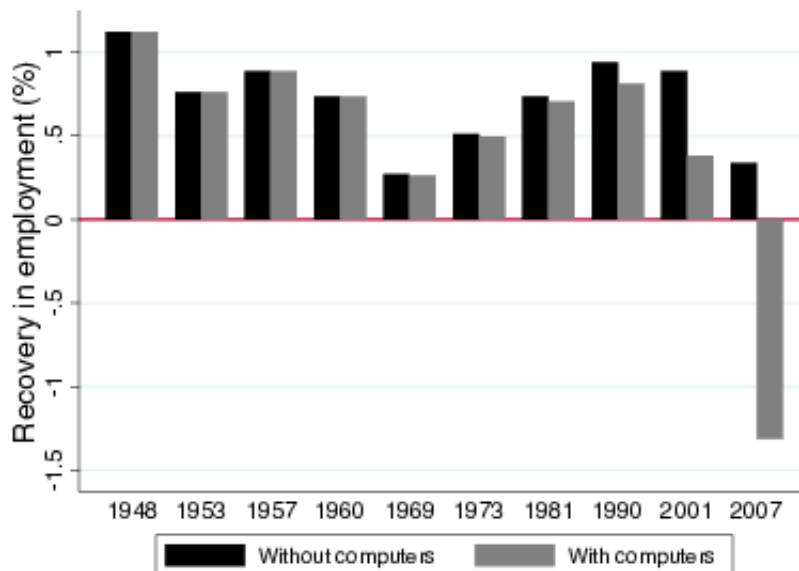


Figure 6: The model with a falling price of computers predicts weaker recoveries of employment after recent recessions compared to the model where the price of computers is constant at its initial level.

Details: recovery of employment for a given recovery of output of 5%, as in Figure 1.

5 Testing the model: labor demand versus labor supply

This section tests the labor demand mechanism in this paper against the labor supply mechanism of Jaimovich and Siu (2012b). It uses the identification strategy in Andersen et al. (2012): a state’s density of lightning flashes causes power surges in the electricity network, damages chips in micro-computers, and predicts computer adoption and productivity growth after 1995 but not before. This section examines the effect of this instrument on the two main reasons for unemployment—layoffs or quits.

Unemployment data comes from the March supplement of the Current Population Survey between 1988 and 2014, which asked workers if they were unemployed because they were “job loser / on layoff,” i.e. a labor demand mechanism, or “job leaver,” i.e. a labor supply mechanism.²⁵ The probability of unemployment due to layoff or quit in state k and year t is the frequency count across workers indexed by i :

$$\mathbb{P}(\text{reason}_{k,t} | \text{unemployed}_{k,t}) = \frac{\#\{i | \text{reason}_{i,k,t} \ \& \ \text{unemployed}_{i,k,t}\}}{\#\{i | \text{unemployed}_{i,k,t}\}}, \quad \text{reason} \in \{\text{layoff}, \text{quit}\}.$$

The instrument of lightning flashes is the number of lightning strikes from US weather stations by state, averaged across the years 1995-2000 and divided by area. Lightning flashes hinder the adoption of computer technology, so this section uses $\text{flash}'_k = -\text{flash}_k$ in the right-hand side of the regressions so the estimated coefficient has the interpretation of the causal effect of computer adoption.

The main specification is the following cross-sectional, reduced-form regression:

$$\mathbb{P}(\text{reason}_{k,t} | \text{unemployed}_{k,t}) = \alpha_t + \beta_{\text{reason},t} \text{flash}'_k + \epsilon_{k,t}, \quad \text{reason} \in \{\text{layoff}, \text{quit}\} \quad (5.1)$$

estimated by OLS for the 49 mainland states (including Washington DC), for a given year

²⁵The four other reasons are “Other job loser”, “Temporary job ended”, “Re-entrant”, and “New entrant.” I use the person’s sampling weight $wtfinl$ to aggregate at the state- and year-level.

and reason for unemployment. The left-hand side conditions on unemployment and controls for the aggregate state of the economy.

Figure 7 plots the time-varying coefficients $\beta_{\text{layoff},t}$ and standard errors from the regression (5.1) with layoffs as the source of unemployment. The coefficient is always statistically significant after 1994, implying that a state with lower cost of computer adoption has a higher frequency of layoffs as the reason for unemployment. The coefficient is also economically significant: at an average of 0.0028 across years, moving from the 10th percentile of (negative) lightning density to the 90th percentile raises the probability of layoff by 9 percentage points, conditional on unemployment.

In contrast, Figure 8 plots the resulting coefficients of regression equation (5.1) for quits as the reason for unemployment. The coefficient is almost never statistically significant. When the coefficient is statistically significant, the sign has the unexpected sign: higher computer adoption costs *decreases* the probability of quits as the source of unemployment.²⁶

These results are robust to alternative specifications, such as running the OLS regression weighted by the number of unemployed workers in each year-state cell and to including “Other job loser” in the layoff category. They are qualitatively similar when further conditioning on routine-cognitive occupations.²⁷ It seems intuitive that workers don’t quit into unemployment when times are bad, i.e. a recession is not a good time for a worker to separate from a firm. Overall, these results suggest that the labor demand mechanism in this paper is a better illustration than the labor supply mechanism of Jaimovich and Siu (2012b) for

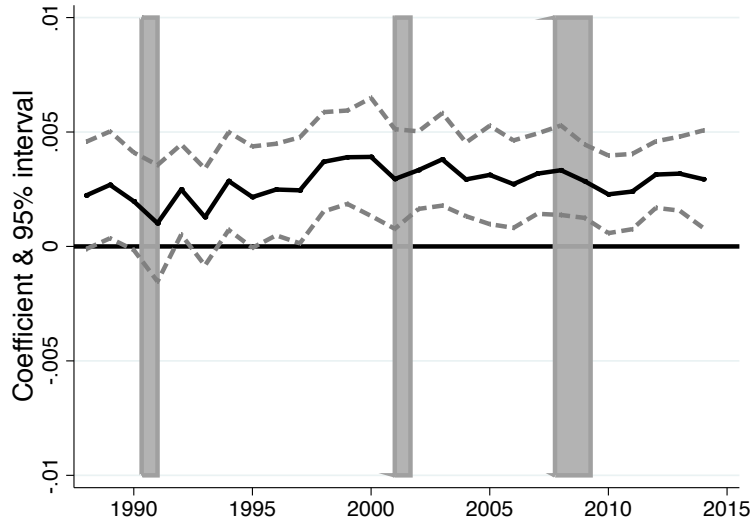


Figure 7: Time-varying coefficients for the effect of lightning flashes on layoffs as the source of unemployment: computer adoption increases the probability of layoffs, conditional on unemployment.

Note: the coefficient is the OLS estimator from independent cross-sectional regressions by state of the probability of having been laid off, conditional on unemployment, on the negative of lightning flash density. The dashed lines are 95% confidence intervals. Source: see text.

the forces driving unemployment in the data.

6 Conclusion

This paper studies the link between computers and the behavior of the labor market in the medium-term and the short-term. The model matches three structural changes in the labor market since the 1980s: a shift away from routine occupations, a productivity speed-up, and a decline in the labor share of income. The model also matches two cyclical changes: routine job losses concentrated in recessions and jobless recoveries. The labor demand mechanism in this paper finds more support in the data than the labor supply mechanism in Jaimovich

²⁶ Elsby et al. (2009) document that quit rates are a third to a fourth smaller than layoff rates; accordingly, the average coefficient is an order of magnitude smaller for quits compared to layoffs.

²⁷Routine-cognitive occupations are the most vulnerable to computer replacement, but the statistical precision of the coefficients is smaller. A state-year cell contains, on average, 109 observations of unemployed workers, with 16 job losers and 8 job leavers. Conditioning on routine-cognitive occupation, these numbers drop significantly: each cell has on average 25 unemployed workers, with 2 job losers and 3 job leavers.

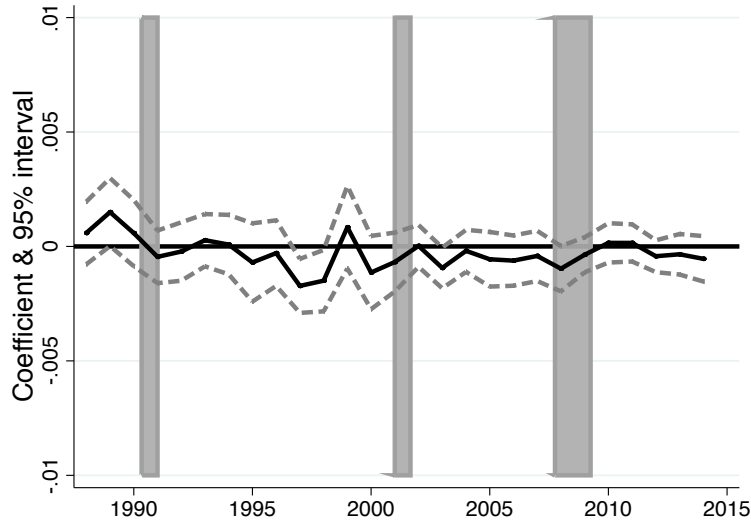


Figure 8: Time-varying coefficients for the effect of lightning flashes on quits as the source of unemployment: computer adoption does not affect the probability of quits, conditional on unemployment.

Details: see Figure 7.

and Siu (2012b).

The model predicts that these labor market changes should occur in all countries, since the decline in the price of computers was a global trend. Using industry- and country-level data, Michaels et al. (2010) find that industries that invest more in computers also increase demand for nonroutine, highly-educated workers. Furthermore, countries that invest more in computers, such as the US, the UK, Sweden, and Japan, have also experienced jobless recoveries since the 1980s. Countries may invest differently in computers because, as Bloom et al. (2007) suggested, computers complement the managerial practices in the United States but not in Europe. The model could capture these differences across countries i with a parameter λ_i in front of computer capital $K_{C,i,t}$, which would affect the relative price of computers and could account for the distinct timing of computer adoption and labor market changes. Using the model to fit the cross-country evidence is left for future research.

Are jobless recoveries the new norm? Jaimovich and Siu (2012a) think so, but this paper suggests a qualified conclusion: if the decrease in the price of computers slows down before the next recession, the following recovery may well be “jobful.”

If the next recession occurs before this slowdown, the recovery may be jobless and this paper suggests a new tradeoff for monetary authorities during jobless recoveries. In a more general model with sticky prices, the interest rate is the cost of present consumption and

also the cost of capital. If the monetary authority keeps interest rates low, it encourages firms to invest in computer capital instead of recruiting routine labor services; if it raises interest rates, there may be no recovery at all. An analysis of this tradeoff is also left for future research.

References

- Aghion, P. and G. Saint-Paul (1998). Virtues of Bad Times: Interaction Between Productivity Growth and Economic Fluctuations. *Macroeconomic Dynamics* 2, 322–344.
- Andersen, T. B., J. Bentzen, C.-J. Dalgaard, and P. Selaya (2012, November). Lightning, it diffusion, and economic growth across u.s. states. *Review of Economics and Statistics* 94(4), 903–924.
- Andolfatto, D. and G. MacDonald (2004). Jobless recoveries. *EconWPA Macroeconomics Paper (accessed 13 April 2011)*.
- Antràs, P. (2004). Is the US aggregate production function Cobb-Douglas? New estimates of the elasticity of substitution. *The BE Journal of Macroeconomics* 4(1), 1–34.
- Autor, D. (2010, April). The Polarization of Job Opportunities in the U.S. Labor Market: Implications for Employment and Earnings. *Center for American Progress (accessed 18 October 2011)*.
- Autor, D. and D. Dorn (2009). The Growth of Low Skill Service Jobs and the Polarization of the US Labor Market. *NBER Working paper 15150*.
- Autor, D., L. Katz, and M. Kearney (2006, May). The Polarization of the U.S. Labor Market. *American Economic Review Papers and Proceedings* 96(2), 189–194.
- Autor, D., F. Levy, and R. Murnane (2003, November). The Skill Content of Recent Technological Change: An Empirical Exploration. *The Quarterly Journal of Economics* 118(4), 1279–1333.
- Autor, D. H., L. F. Katz, and M. S. Kearney (2008, May). Trends in u.s. wage inequality: Revising the revisionists. *Review of Economics and Statistics* 90(2), 300–323.
- Bárány, Z. L. and C. Siegel (2018). Job polarization and structural change. *American Economic Journal: Macroeconomics*, 10(1): 57-89.
- Basu, S., J. Fernald, and M. Shapiro (2001). Productivity Growth in the 1990s: Technology, Utilization, or Adjustment? *NBER Working Paper 8359* 55(1), 117–165.

- Bentolila, S. and G. Saint-Paul (1994). A model of labor demand with linear adjustment costs. *Labour Economics* 1, 303–326.
- Berger, D. (2012). Countercyclical Restructuring and Jobless Recoveries. *Manuscript, Yale University (accessed 7 November 2012)*.
- Berndt, E. and N. Rappaport (2001). Price and Quality of Desktop and Mobile Personal Computers: A Quarter-Century Historical Overview. *The American Economic Review* 91(2), 268–273.
- Blanchard, O., W. Nordhaus, and E. Phelps (1997). The Medium Run. *Brookings Papers on Economic Activity* 1997(2), 89–158.
- Blatter, M., S. Muehleemann, and S. Schenker (2012). The costs of hiring skilled workers. *European Economic Review* 56(1), 20–35.
- Bloom, N., R. Sadun, and J. V. Reenen (2007). Americans do I.T. better: US multinationals and the productivity miracle. *NBER Working Paper 13085*.
- Bresnahan, T. and M. Trajtenberg (1995). General Purpose Technologies: 'Engines of growth'? *Journal of Econometrics* 65, 83–108.
- Brynjolfsson, E. and L. Hitt (1994). New evidence on the returns to Information Systems. *Sloan School Working Paper 162*.
- Caballero, R. and M. Hammour (1994). The cleansing effect of recessions. *The American Economic Review* 84(5), 1350–1368.
- Chodorow-Reich, G. (2014, January). The Employment Effects of Credit Market Disruptions: Firm-level Evidence from the 2008-9 Financial Crisis. *The Quarterly Journal of Economics* 129(1), 1–59.
- Conlon, C. (2010). A Dynamic Model of Costs and Margins in the LCD TV Industry. *Manuscript, Yale University (accessed 28 March 2012)*.
- David, P. (1990, May). The Dynamo and the Computer: An Historical Perspective on the Modern Productivity Paradox. *The American Economic Review* 80(2), 355–361.
- Del Boca, A. and P. Rota (1998). How much does hiring and firing cost? Survey evidence from a sample of Italian firms. *Labour* 12(3), 427–449.
- Elsby, M. W. L., R. Michaels, and G. Solon (2009). The ins and outs of cyclical unemployment. *American Economic Journal: Macroeconomics* 1(1), 84–110.
- Field, A. (2011). *A Great Leap Forward: 1930s Depression and U.S. Economic Growth*. Yale University Press.

- Galí, J., F. Smets, and R. Wouters (2012). Slow recoveries: a structural interpretation. *Journal of Money, Credit and Banking* 44(2), 9–30.
- Goldin, C. and L. Katz (2007). Long-Run Changes in the Wage Structure: Narrowing, Widening, Polarizing. *Brookings Papers on Economic Activity* 2007(2), 135–165.
- Goos, M. and A. Manning (2007). Lousy and Lovely Jobs: The Rising Polarization of Work in Britain. *The Review of Economics and Statistics* 89(1), 118–133.
- Goos, M., A. Manning, and A. Salomons (2014). Explaining job polarization: Routine-biased technological change and offshoring. *American Economic Review* 104(8), 2509–2526.
- Gordon, R. (1993). The Jobless Recovery: Does it signal a new era of productivity-led growth? *Brookings Papers on Economic Activity* 3(1), 271–316.
- Greenwood, J., Z. Hercowitz, and G. Huffman (1988, June). Investment, Capacity Utilization, and the Real Business Cycle. *The American Economic Review* 78(3), 402–417.
- Hamermesh, D. (1993). *Labor Demand*. Princeton University Press.
- Helpman, E. and M. Trajtenberg (1994). A time to sow and a time to reap: Growth based on general purpose technologies. *NBER Working paper 4854*.
- Jaimovich, N. and S. Rebelo (2009). Can news about the future drive the business cycle? *American Economic Review* 99(4), 1097–1118.
- Jaimovich, N. and H. Siu (2012a, November). Jobless recoveries and the disappearance of routine occupations. VoxEU.
- Jaimovich, N. and H. Siu (2012b). The Trend is the Cycle: Job Polarization and Jobless Recoveries. *NBER Working Paper 18334*.
- Jones, C. (2011). *Macroeconomics*. W. W. Norton and Company.
- Jorgenson, D. (2001). Information technology and the US economy. *The American Economic Review* 91(1), 1–32.
- Jovanovic, B. and P. L. Rousseau (2005). General Purpose Technologies. In P. Aghion and S. N. Durlauf (Eds.), *Handbook of Economic Growth*, pp. 1181–1224. Elsevier.
- Kaldor, N. (1961). *The Theory of Capital*, Chapter Capital Accumulation and Economic Growth, pp. 177–222. St. Martins Press.
- Karabarbounis, L. and B. Neiman (2013, June). The Global Decline of the Labor Share. *NBER Working Paper 19136*.
- Karabarbounis, L. and B. Neiman (2014, October). Capital depreciation and labor shares around the world: Measurement and implications. *NBER working paper 20606*.

- Keane, M. (2011). Labor Supply and Taxes: A Survey. *Journal of Economic Literature* 49(4), 961–1075.
- Krusell, P., L. Ohanian, J.-V. Rios-Rull, and G. Violante (2000). Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis. *Econometrica* 68(5), 1029–1053.
- Kydland, F. and E. Prescott (1982, November). Time to Build and Aggregate Fluctuations. *Econometrica* 50(6), 1345–1370.
- Maliar, L. and S. Maliar (2015, March). Merging simulation and projection approaches to solve high-dimensional problems with an application to a new Keynesian model. *Quantitative Economics* 6(1), 1–47.
- Maliar, L., S. Maliar, J. Taylor, and I. Tsener (2015, May). A Tractable Framework for Analyzing a Class of Nonstationary Markov Models. NBER Working Papers 21155, National Bureau of Economic Research, Inc.
- Michaels, G., A. Natraj, and J. V. Reenen (2010). Has ICT Polarized Skill Demand? Evidence from Eleven Countries over 25 years. *NBER Working Paper 16138*.
- Nordhaus, W. (2007). Two Centuries of Productivity Growth in Computing. *Journal of Economic History* 67(1), 128–159.
- Oliner, S., D. Sichel, and K. Stiroh (2007). Explaining a Productive Decade. *Brookings Papers on Economic Activity* 2007(1), 81–137.
- Rodriguez, F. and A. Jayadev (2013). The Declining Labor Share of Income. *Journal of Globalization and Development* 3(2), 1–18.
- Saint-Paul, G. and S. Bentolila (2003). Explaining Movements in the Labor Share. *Contributions to Macroeconomics* 3(1).
- Schmitt-Grohe, S. and M. Uribe (2012). What’s news in business cycles. *Econometrica* 80(6), 2733–2764.
- Schreft, S., A. Singh, and A. Hodgson (2005). Jobless recoveries and the wait-and-see hypothesis. *Economic Review, Federal Reserve Bank of Kansas City*.
- Sichel, D. (1997). *The Computer Revolution: An Economic Perspective*. The Brookings Institution Press.
- Stokey, N. and R. Lucas (1989). *Recursive Methods in Economic Dynamics*. Harvard University Press.

Data sources

Bureau of Economic Analysis. 2014. “Price Indexes for Private Fixed Investment in Equipment by Type (Table 5.5.4U).” Department of Commerce.

<http://www.bea.gov/itable/> (accessed 16 April 2014).

Bureau of Economic Analysis. 2012. “Private Fixed Investment in Equipment and Software by Type, Table 5.5.5U.” Department of Commerce. <http://www.bea.gov> (accessed 7 September 2012).

Federal Reserve Bank of St. Louis. 2013. “Federal Reserve Economic Database.” Federal Reserve Bank. <http://research.stlouisfed.org/fred2> (accessed 16 April 2014).

King, Miriam, Steven Ruggles, J. Trent Alexander, Sarah Flood, Katie Genadek, Matthew B. Schroeder, Brandon Trampe, and Rebecca Vick. 2010. “Integrated Public Use Microdata Series, Current Population Survey: Version 3.0.” Minneapolis: University of Minnesota. <http://cps.ipums.org> (accessed 18 August 2012).

O*Net resource center. 2012. “Occupational Information Network.” Department of Labor. <http://www.onetcenter.org> (accessed 18 August 2012).

Vaisala. Flash Counts By County, 1995-2000 (electronic correspondence).

A Equilibrium of the model

Denote $\nu_{C,t}$ and $\nu_{NC,t}$ the Lagrange multipliers of the capital accumulation constraints (equations 2.2 and 2.3), and μ_t the multiplier of the budget constraint (equation 2.4). Denote $H_{NR,t}$ and $H_{R,t}$ the recruitment of nonroutine and routine labor services, with constraints $L_{J,t+1} \leq L_{J,t} + H_{J,t}$ and $H_{J,t} \geq 0$ for $J = NR, R$. The first constraint implies that increases in employment have to come from hiring. The second constraint implies that hiring is never negative. (If hiring were negative, the firm would receive subsidies for dismissing labor services.) Denote $\psi_{J,t}$ the multiplier on the first constraint and $\vartheta_{J,t}$ the multiplier on the second constraint. Denote $\iota_{C,t}$ and $\iota_{NC,t}$ the multipliers on the positivity constraint for investment: $I_{C,t} \geq 0$, $I_{NC,t} \geq 0$.

The first-order conditions of the household’s program are:

$$\begin{aligned} \left(C_t - X_t \frac{\varepsilon}{1 + \varepsilon} L_t^{\frac{1+\varepsilon}{\varepsilon}} \right)^{-1} &= \mu_t \\ X_t L_t^{\frac{1}{\varepsilon}} &= w_t \\ \mu_t r_{NC,t} &= \theta^{-1} \nu_{NC,t-1} - \nu_{NC,t} (1 - \delta_{NC}) \\ \mu_t r_{C,t} &= \theta^{-1} \nu_{C,t-1} - \nu_{C,t} (1 - \delta_C) \\ \nu_{NC,t} &= \mu_t - \iota_{NC,t} \\ \nu_{C,t} &= \mu_t \exp(-b_t) - \iota_{C,t} \end{aligned}$$

The household’s subjective discount factor, inherited by the firm, is $D_{0,t} = \theta^t \mu_t / \mu_0$. The

household's program has four complementarity slackness conditions:

$$\begin{aligned} 0 &= I_{NC,t} \iota_{NC,t} = I_{C,t} \iota_{C,t}, \\ 0 &= \nu_{NC,t} ((1 - \delta_{NC}) K_{NC,t} + I_{NC,t} - K_{NC,t+1}) = \nu_{C,t} ((1 - \delta_C) K_{C,t} + I_{C,t} - K_{C,t+1}). \end{aligned}$$

For ease of notation, this appendix uses $\rho = (\sigma - 1) / \sigma$. The first-order conditions of the firm imply:

$$\begin{aligned} MPL_{NR,t} &= \beta A_t K_{NC,t}^\alpha L_{NR,t}^{\beta-1} (K_{C,t}^\rho + L_{R,t}^\rho)^{\gamma/\rho} = w_t + \frac{D_{0,t-1}}{D_{0,t}} \psi_{NR,t-1} - \psi_{NR,t}, \\ MPL_{R,t} &= \gamma A_t K_{NC,t}^\alpha L_{R,t}^{\rho-1} L_{NR,t}^\beta (K_{C,t}^\rho + L_{R,t}^\rho)^{\frac{\gamma}{\rho}-1} = w_t + \frac{D_{0,t-1}}{D_{0,t}} \psi_{R,t-1} - \psi_{R,t}, \\ MPK_{NC,t} &= \alpha A_t K_{NC,t}^{\alpha-1} L_{NR,t}^\beta (K_{C,t}^\rho + L_{R,t}^\rho)^{\gamma/\rho} = r_{NC,t}, \\ MPK_{C,t} &= \gamma A_t K_{C,t}^{\rho-1} K_{NC,t}^\alpha L_{NR,t}^\beta (K_{C,t}^\rho + L_{R,t}^\rho)^{\frac{\gamma}{\rho}-1} = r_{C,t}, \\ \vartheta_{NR,t} &= c_{NR} - \psi_{NR,t}, \\ \vartheta_{R,t} &= c_R - \psi_{R,t}, \end{aligned}$$

where MPF is the marginal product of factor F . The firm makes zero intertemporal profits but it may make positive or negative profits in each period, reverted to or financed by the household. The firm's program has two complementarity slackness conditions:

$$\begin{aligned} \vartheta_{NR,t} H_{NR,t} &= \psi_{NR,t} (L_{NR,t} + H_{NR,t} - L_{NR,t+1}) = 0, \\ \vartheta_{R,t} H_{R,t} &= \psi_{R,t} (L_{R,t} + H_{R,t} - L_{R,t+1}) = 0. \end{aligned}$$

The set of equilibrium conditions also includes the physical constraints of the model (equations 2.2-2.5), the labor market clearing, and the following transversality conditions:

$$\lim_{t \rightarrow \infty} D_{0,t} K_{NC,t} = \lim_{t \rightarrow \infty} D_{0,t} K_{C,t} = 0.$$

For computational reasons, the numerical solution truncates the horizon at $T < \infty$. An equilibrium, solved by AMPL (A Mathematical Programming Language) is a set of $19 \times T$ variables (consumption C_t , capital stocks $K_{C,t}$ and $K_{NC,t}$, investments $I_{C,t}$ and $I_{NC,t}$, employment quantities L_t , $L_{NR,t}$ and $L_{R,t}$, output Y_t , rental rates $r_{C,t}$ and $r_{NC,t}$, wages w_t , Lagrange multipliers $\nu_{C,t}$, $\nu_{NC,t}$, μ_t , $\psi_{NR,t}$, $\vartheta_{NR,t}$, $\iota_{C,t}$, $\iota_{NC,t}$) solving $19 \times T$ equations (capital accumulation constraints (2.2-2.3), budget constraint (2.4), production function (2.5), labor market equilibrium, six optimality conditions for the household, five optimality conditions for the firm, and three complementarity slackness conditions).

The numerical solution replaces some of these equations with boundary conditions. Eight equations are intertemporal and involve quantities at times t and $t + 1$: the two capital accumulation constraints, the two labor accumulation constraints, the two first-order conditions for the firm on labor, and the two first-order conditions for the household on capital accumulation. The equilibrium has $T - 1$ of these equations, with 8 equations missing from

the total set. These eight equations are replaced with boundary conditions for the two types of capital and the two types of labor at time 1 and time T , equal to their values in the initial or final steady-state. The steady-state is a set of time-independent variables solving these equations when the outside variables (A_t, b_t, X_t) or (b_t, X_t, Y_t) are fixed at their level at time 1 or time T . To ensure that these boundary conditions play a minimal role, the calibration includes a buffer of 20 time periods at the beginning and 60 time periods at the end, where the outside variables equal their initial or final values, e.g. $A_t = A_1$ for $t \leq 20$ and $b_t = b_T$ for $t \geq T - 60$.

B Characterization of the model

An equilibrium of this model exists as long as the labor supply of the household is bounded above. This assumption is used only in the theoretical setting and never binds numerically.

Lemma 9. *If the labor supply of the household is bounded above, $L_t \leq \bar{L}$, an equilibrium exists and it is unique.*

Proof. Given that this model has no market failures, the market equilibrium coincides with the optimum of a benevolent social planner who maximizes the household's utility:

$$\max \sum_{t=0}^{\infty} \theta^t \log \left(C_t - X_t \frac{\varepsilon}{1 + \varepsilon} L_t^{\frac{1+\varepsilon}{\varepsilon}} \right),$$

subject to the physical constraints in equations (2.3-2.2), (2.5-2.5), the labor market clearing, and to the following resource constraint (implied by the definition of profits, the budget constraint, and the labor market equilibrium):

$$Y_t = C_t + I_{NC,t} + \exp(-b_t) I_{C,t} + c_{NR} (L_{NR,t+1} - L_{NR,t})^+ + c_R (L_{R,t+1} - L_{R,t})^+.$$

The Bellman formulation for the planner's problem uses seven control variables and five state variables $S_t = \{K_{NC,t}, K_{C,t}, L_{NR,t}, L_{R,t}, t\}$:

$$V(S_t) = \max_{C_t, H_{NR,t}, H_{R,t}, I_{C,t}, I_{NC,t}} \left\{ \log \left(C_t - X_t \frac{\varepsilon}{1 + \varepsilon} L_t^{\frac{1+\varepsilon}{\varepsilon}} \right) + \theta V(S_{t+1}) \right\},$$

subject to the same physical constraints.

The contraction mapping for a Bellman operator requires three Blackwell conditions. First, the set of controls is bounded: hiring variables are bounded above by maximum labor supply \bar{L} and quantity variables of consumption and investment are bounded by production Y_t , which is set by the four inputs as state variables. Both the disutility from labor supply and the utility from consumption are bounded. The Bellman operator maps the space of bounded functions into itself.

The remaining two conditions, monotonicity and discounting, follow from the Bellman formulation of the problem with a discount parameter θ . The contraction mapping theorem

guarantees existence and uniqueness of the equilibrium of the model Stokey and Lucas (see 1989, page 54).

□

The model has an asymptotic balanced growth path, consistent with the “Kaldor facts” of a constant interest rate and a constant capital-output ratio (Kaldor, 1961). The following lemma characterizes the behavior of the asymptotic balanced growth path where employment is constant and all other quantities, aside from employment, grow at the same rate.

Lemma 10. *Consider the limiting economy, where TFP grows at rate $g_A > 0$, b_t tends to \bar{b} , the marginal utility from consumption declines at rate g_μ , the capital stocks grow at rate $g_{K_{NC}}$ and g_{K_C} , and the labor supply shifter grows at rate $g_X = g_A/\beta$. Then employment is constant and consumption, output, and all quantities other than employment grow at rate g_A/β .*

Proof. In the limiting balanced growth path, where the capital stocks grow at constant rates, investment is positive and the Lagrange multipliers on investment are zero: $\nu_{NC,t} = \nu_{C,t} = 0$. The Lagrange multipliers on capital accumulation are linked to the marginal utility μ_t from consumption:

$$\nu_{NC,t} = \mu_t, \quad \nu_{C,t} = \mu_t \exp(-b_t).$$

The equilibrium rental rates of capital are constant:

$$\begin{aligned} r_{NC,t} &= \theta^{-1} \frac{\mu_{t-1}}{\mu_t} - 1 + \delta_{NC} \rightarrow \theta^{-1} (1 + g_\mu)^{-1} - 1 + \delta_{NC}, \\ r_{C,t} &= \theta^{-1} \frac{\mu_{t-1}}{\mu_t} \exp(-b_{t-1}) - \exp(-b_t) (1 - \delta_C) \rightarrow \exp(-\bar{b}) (\theta^{-1} (1 + g_\mu)^{-1} - 1 + \delta_C). \end{aligned}$$

The firm’s limiting subjective one-period discount factor also converges: $D_{0,t-1}/D_{0,t} = \mu_{t-1}/\theta\mu_t \rightarrow \theta^{-1} (1 + g_\mu)^{-1}$. The factor price frontier, implied by the firm’s first-order conditions, is:

$$\underbrace{\alpha^\alpha \beta^\beta \gamma^\gamma A_t}_{\rightarrow \infty} = r_{NC,t}^\alpha w_t^\beta \tau_{NR,t}^\beta (r_{C,t}^{1-\sigma} + w_t^{1-\sigma} \tau_R^{1-\sigma})^{\frac{\gamma}{1-\sigma}}, \quad \tau_{J,t} = 1 + \frac{D_{0,t-1}}{D_{0,t}} \frac{\psi_{J,t-1}}{w_t} - \frac{\psi_{J,t}}{w_t}.$$

The left-hand side of the factor price frontier diverges. The wage cannot converge to zero, otherwise the right-hand side of the factor price frontier converges to zero. So the wage is bounded away from zero. On the right-hand side, the two rental rates of capital and the one-period discount factor converge. The multipliers $\{\psi_{NR,t}, \psi_{R,t}\}$ are bounded between 0 and $\{c_{NR}, c_R\}$, the wage is bounded away from zero, and the one-period discount factor converges, so the terms $\tau_{J,t}$ are bounded. All terms on the right-hand side converge or are bounded, except for wages w_t . Therefore, wages also diverge and grow indefinitely at a rate implied by the limiting factor price frontier: $g_w = g_A/\beta$.

Given constant rental rates of capital and unbounded wages, the limiting capital-output ratios are constant:

$$\frac{K_{NC,t}}{Y_t} = \frac{\alpha}{r_{NC,t}} \rightarrow \frac{\alpha}{r_{NC}}, \quad \frac{K_{C,t}}{Y_t} = \frac{\gamma}{r_{C,t}} \left(1 + \left(\frac{w_t}{r_{C,t}} \right)^{1-\sigma} \right)^{-1} \rightarrow \frac{\gamma}{r_C}.$$

The labor supply equation from the household is $X_t L_t^{\frac{1}{\beta}} = w_t$. For a balanced growth path with constant employment, the growth in the disutility of labor supply has to verify $g_X = g_w = g_A/\beta$.

As wages grow indefinitely, the relative cost of computer capital decreases to zero and employment reallocates entirely from routine to nonroutine labor services $L_{NR,t} \rightarrow L$ and $L_{R,t} \rightarrow 0$. The limiting production function is a three-factor Cobb-Douglas:

$$Y_t \rightarrow A_t K_{NC,t}^\alpha L_{NR,t}^\beta K_{I,t}^\gamma,$$

which implies the following equation between limiting growth rates $g_Y = g_A + \alpha g_{K_{NC}} + \beta g_L + \gamma g_{K_I}$. Using the constant capital-output ratios and the limiting growth rate of employment, the growth rate of output is $g_Y = g_A + \alpha g_Y + \gamma g_Y = g_A/\beta$.

At the limit, the investment-capital ratios are constant:

$$\frac{I_{J,t}}{K_{J,t}} = \frac{K_{J,t+1}}{K_{J,t}} - (1 - \delta_J) \rightarrow g_{K_J} + \delta_J = g_Y + \delta_J, \quad J = C, NC.$$

Therefore, the investment-output ratios are also constant and the two types of investment grow at rate $g_Y = g_A/\beta$. The resource constraint implies that consumption tends to a constant share of output:

$$\begin{aligned} \frac{C_t}{Y_t} &= 1 - \frac{I_{NC,t}}{Y_t} - \exp(-b_t) \frac{I_{C,t}}{Y_t} \\ &= 1 - \frac{I_{NC,t}}{K_{NC,t}} \frac{K_{NC,t}}{Y_t} - \exp(-b_t) \frac{I_{C,t}}{K_{C,t}} \frac{K_{C,t}}{Y_t} \\ &\rightarrow 1 - \left(\delta_{NC} + \frac{g_A}{\beta} \right) \frac{\alpha}{r_{NC}} - \exp(-\bar{b}) \left(\delta_C + \frac{g_A}{\beta} \right) \frac{\gamma}{r_C}. \end{aligned}$$

Therefore, consumption grows at the same rate as output, and all quantities grow at the same rate, as well as the wage: $g_C = g_Y = g_{K_I} = g_{K_{NC}} = g_{I_I} = g_{I_{NC}} = g_w = \frac{g_A}{\beta}$.

□

C Proofs in the special case of the model

Proof of lemma 3.1. This proof omits the time index t . For the equilibrium condition, note that the resource constraint on the product market and the household's budget constraint imply zero profits for the firm. Denoting Π_j the indexed product operator (different from the logarithm π of labor productivity), consider a multi-factor Cobb-Douglas production function, $Y = A \prod_j F_j^{\alpha_j}$, with constant returns to scale $\sum \alpha_j = 1$. Denote the marginal cost of each factor F_j with mc_j . Optimization of this production function implies constant factor shares: $F_j = \alpha_j Y / mc_j$. Raising to the power α_j and multiplying over j yields:

$$\frac{Y}{A} = \prod_j F_j^{\alpha_j} = \prod_j \left(\frac{\alpha_j}{mc_j} \right)^{\alpha_j} \times Y^{\sum \alpha_j} \implies \frac{1}{A} \prod_j \left(\frac{mc_j}{\alpha_j} \right)^{\alpha_j} = 1.$$

The marginal cost of the first two factors, K_{NC} and L_{NR} , is 1 and w . The marginal cost of the third Cobb-Douglas factor, the Constant-Elasticity-of-Substitution aggregate, requires more detail. Consider a firm that is selling the third Cobb-Douglas factor at marginal cost mc_3 to maximize profits:

$$\max_{K_C, L_R} mc_3 (K_C^\rho + L_R^\rho)^{\frac{1}{\rho}} - r_C K_C - w L_R.$$

The ratio of first-order conditions on capital K_C and labor L_R imply:

$$\left(\frac{K_C}{L_R} \right)^{\rho-1} = \frac{r_C}{w} \implies \frac{K_C}{L_R} = \left(\frac{w}{r_C} \right)^\sigma$$

The first-order condition for labor implies $mc_3 (K_C^\rho + L_R^\rho)^{\frac{1}{\rho}-1} L_R^{\rho-1} = w$. Rearrange this expression, use $\sigma\rho = \sigma - 1$ and the solution for computer capital relative to employment in routine occupations to obtain:

$$\begin{aligned} mc_3 &= \left(1 + \left(\frac{K_C}{L_R} \right)^\rho \right)^{1-\frac{1}{\rho}} w = \left(1 + \left(\frac{w}{r_C} \right)^{\rho\sigma} \right)^{\frac{1}{1-\sigma}} w \\ mc_3 &= (1 + w^{\sigma-1} r_C^{1-\sigma})^{\frac{1}{1-\sigma}} (w^{1-\sigma})^{\frac{1}{1-\sigma}} = (r_C^{1-\sigma} + w^{1-\sigma})^{\frac{1}{1-\sigma}} \end{aligned}$$

The zero profit condition of the three factor Cobb-Douglas production function is finally

$$\frac{1}{A} \left(\frac{1}{\alpha} \right)^\alpha \left(\frac{w}{\beta} \right)^\beta \left(\frac{(r_C^{1-\sigma} + w^{1-\sigma})^{\frac{1}{1-\sigma}}}{\gamma} \right)^\gamma = 1.$$

This equation is the equilibrium condition for the wage, where the marginal cost of production equals the marginal revenue. The left-hand side is strictly increasing in w , equals 0 for $w = 0$ and tends to infinity for $w \rightarrow \infty$. Therefore, the wage that verifies the equation is unique.

□

Proof of proposition 5. The routine share of employment is:

$$\frac{L_{R,t}}{L_t} = \frac{L_{R,t}}{L_{NR,t} + L_{R,t}} = \left(1 + \frac{L_{NR,t}}{L_{R,t}}\right)^{-1} = \left(1 + \frac{\beta r_{C,t}^{1-\sigma} + w_t^{1-\sigma}}{\gamma w_t^{1-\sigma}}\right)^{-1},$$

where the third equality uses the first-order conditions for the firm. At the limit $b_t \rightarrow -\infty$, the wage tends to a lower bound \underline{w} pinned down by the factor price frontier. At the limit $b_t \rightarrow \infty$, the factor price frontier implies that the wage diverges. The limiting values of the routine share of employment are

$$\lim_{b_t \rightarrow -\infty} \frac{L_{R,t}}{L_t} = \lim_{r_{C,t} \rightarrow \infty} \frac{L_{R,t}}{L_t} = \left(1 + \frac{\beta}{\gamma}\right)^{-1} = \frac{\gamma}{\beta + \gamma}, \quad \lim_{b_t \rightarrow \infty} \frac{L_{R,t}}{L_t} = \lim_{r_{C,t} \rightarrow 0} \frac{L_{R,t}}{L_t} = 0.$$

To compute the impact of the change in the price of computers on the routine share of employment, denote $s_t = \log(L_{R,t}/L_t)$ the logarithm of the routine share of employment. The elasticity of the routine share of employment, after accounting for the effect of b_t on the wage, is negative:

$$\frac{\partial s_t}{\partial b_t} = (1 - \sigma) \beta (\beta + \gamma) \frac{1 + (e^{b_t} w_t)^{(1-\sigma)}}{\left(\beta + (\beta + \gamma) (e^{b_t} w_t)^{(1-\sigma)}\right)^2}.$$

This elasticity is negative: cheaper computers decrease the routine share of employment. The limiting values of the elasticity are as in the text.

□

Proof of proposition 6. Labor productivity $\pi_t = \log(Y_t/L_t)$ is:

$$\pi_t = \log \frac{w_t}{\beta + \gamma (1 + r_{C,t}^{1-\sigma} w_t^{\sigma-1})^{-1}}.$$

where the rental cost of computer capital is $r_{C,t} = \exp(-b_t)$ (see the proof of Proposition 7 for details). The first derivative of labor productivity with respect to b_t is:

$$\frac{\partial \pi_t}{\partial b_t} = \gamma \frac{\beta + (\beta + \gamma) \sigma (e^{b_t} w_t)^{1-\sigma}}{\left(\beta + (\beta + \gamma) (e^{b_t} w_t)^{1-\sigma}\right)^2}$$

At the limit $b_t \rightarrow -\infty$, the wage tends to a finite value \underline{w} , which solves the factor price frontier (3.1) with $r_{C,t} \rightarrow \infty$. The term $(e^{b_t} w_t)^{1-\sigma}$ tends to infinity. Factoring that term in the numerator and the denominator, the numerator tends to $(\beta + \gamma) \sigma$ and the denominator

tends to infinity, so the fraction tends to 0. At the limit $b_t \rightarrow \infty$, the wage grows arbitrarily large and the term $(e^{b_t} w_t)^{1-\sigma}$ tends to zero, so the derivative tends to γ/β . \square

Proof of proposition 7. The labor share of income is:

$$\frac{w_t L_t}{Y_t} = \frac{w_t L_{NR,t}}{Y_t} + \frac{w_t L_{R,t}}{Y_t} = \beta + \gamma \frac{w_t^{\beta+1-\sigma} (r_{C,t}^{1-\sigma} + w_t^{1-\sigma})^{\frac{\gamma}{1-\sigma}-1}}{w_t^\beta (r_{C,t}^{1-\sigma} + w_t^{1-\sigma})^{\frac{\gamma}{1-\sigma}}} = \beta + \gamma \left(1 + \left(\frac{r_{C,t}}{w_t} \right)^{1-\sigma} \right)^{-1}.$$

The cost $r_{C,t}$ of computer capital decreases with time, while the wage w_t increases with time, so the labor share of income unambiguously decreases with time. At the limit $b_t \rightarrow -\infty$, the rental rate $r_{C,t}$ of computers becomes arbitrarily large while the wage converges to \underline{w} , so the labor share of income tends to $\beta + \gamma$. At the limit $b_t \rightarrow \infty$, the rental rate $r_{C,t}$ of computers becomes arbitrarily small, the wage becomes arbitrarily large, so the labor share of income tends to β . \square

Proof of corollary 8. Take the limit $\sigma \rightarrow 1$ in the expressions for the last three proofs. \square

D More aggregate predictions of the model in the data

This subsection relates other predictions of the model that holds in the data. The model predicts that the decline in the labor share of income is entirely due to routine occupations: Figure 9 shows the labor share of income for routine and nonroutine occupations and supports this prediction. Figure 11 shows the path of the labor share in the data and in the model fitting US data. In the data, the labor share decreased 7.5% between the trough of the 1981 recession and the trough of the 2007 recession. The magnitude is similar in the model. Figure 10 shows the path of routine labor services in the model and in the data. The model has a good fit after 2000.

Figure 12 shows that the behavior of computer investment is similar in the model and in the data. The top panel shows the acceleration of the share of computers in fixed investment in the data and in the model. In the data, this share increased 8 percentage points between 1960 and 1980 and 21 percentage points from 1980 to 2000. The bottom panel shows the behavior of computer investment in the data and in the model. This paper explains the acceleration of routinization during recessions and jobless recoveries with computers. Yet, it predicts that computer investment is procyclical instead of accelerating in recessions. The absence of adjustment costs to capital leaves computer investment free to adjust: it falls in recessions and increases in recoveries. The model matches the behavior of computer investment: after

a recession, computer investment simply catches up with its trend, rather than accelerating or increasing to a permanently higher level.

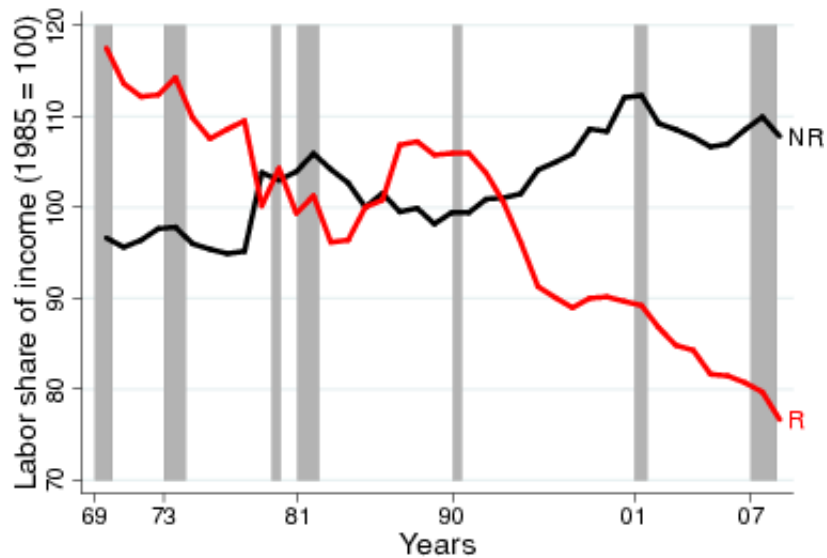


Figure 9: The decline in the labor share of income is entirely due to routine occupations.
Source: Current Population Survey, Occupational Information Network, and Federal Reserve Economic Database. Routine occupations are quartiles 3 and 4, nonroutine occupations are quartiles 1 and 2. The labor share of income for routine occupations is the labor income of routine occupations as a share of total labor income (held constant across the threshold years of 1982, 1992, and 2002), multiplied by the labor share of the nonfarm business sector (series PRS85006173).

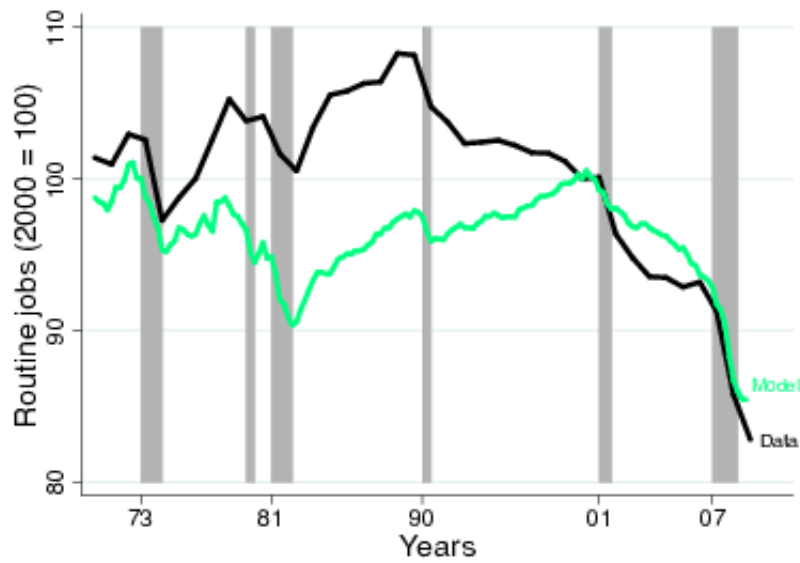


Figure 10: The model matches the decline in employment of routine occupations in the data since 2000.

Source: Current Population Survey, Occupational Information Network, and model simulations. See text for details. Shaded areas are NBER recessions.

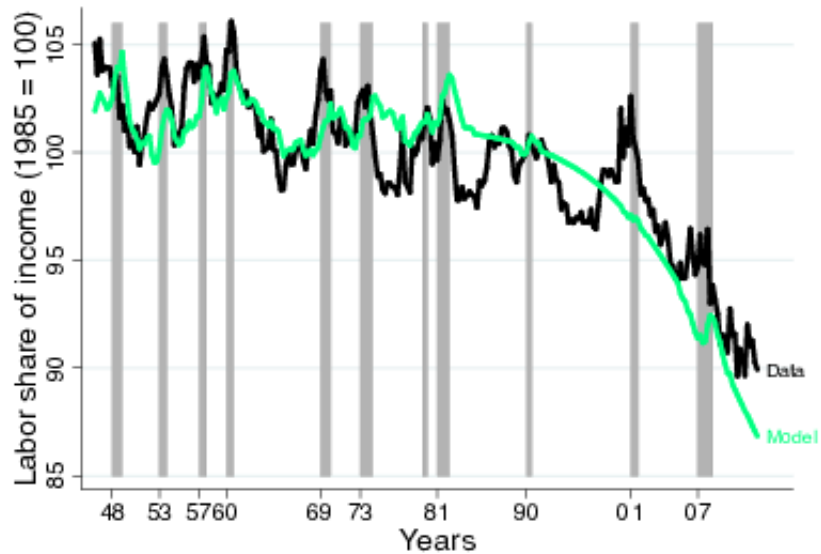


Figure 11: The model matches the path of the labor share of income in the data.

Source: Federal Reserve Economic Database, with labor share of the nonfarm business sector. Shaded areas are NBER recessions.

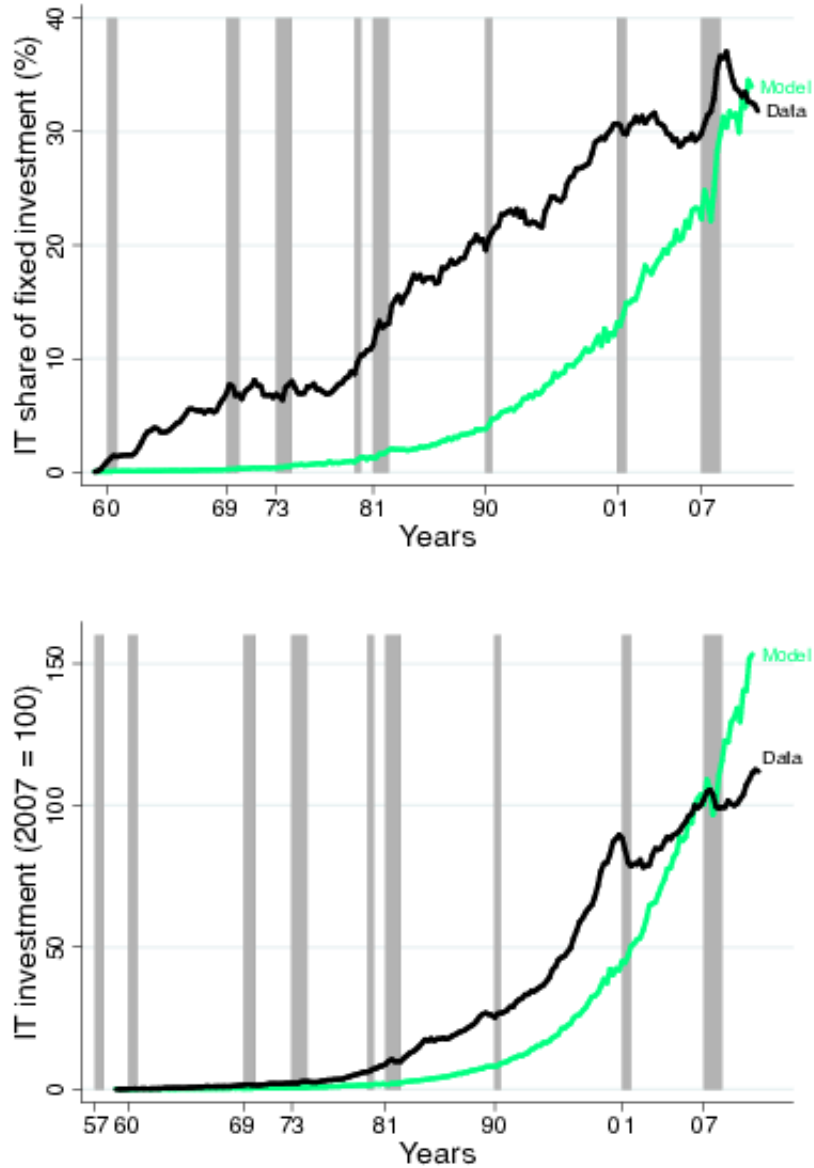


Figure 12: Similar behavior of computer capital in the model and in the data: the share of computers in fixed investment accelerates upward in recent decades (top); after recessions, computer investment returns to trend (bottom).

Data: investment in computers, peripheral equipment, and software, divided by nonresidential fixed investment in equipment (BEA series B935RC0, B985RC0, and B010RC0, from Table 5.5.5U, “Private Fixed Investment in Equipment and Software by Type”). The series in the model and in the data are in nominal terms.